



# Of Mice and Men

4th Workshop on Topological Methods in Data Analysis

University of Heidelberg

20 September 2023

**EPFL**

Project leader: Lida Kanari (Blue Brain Project)



# The topology of neuron morphologies

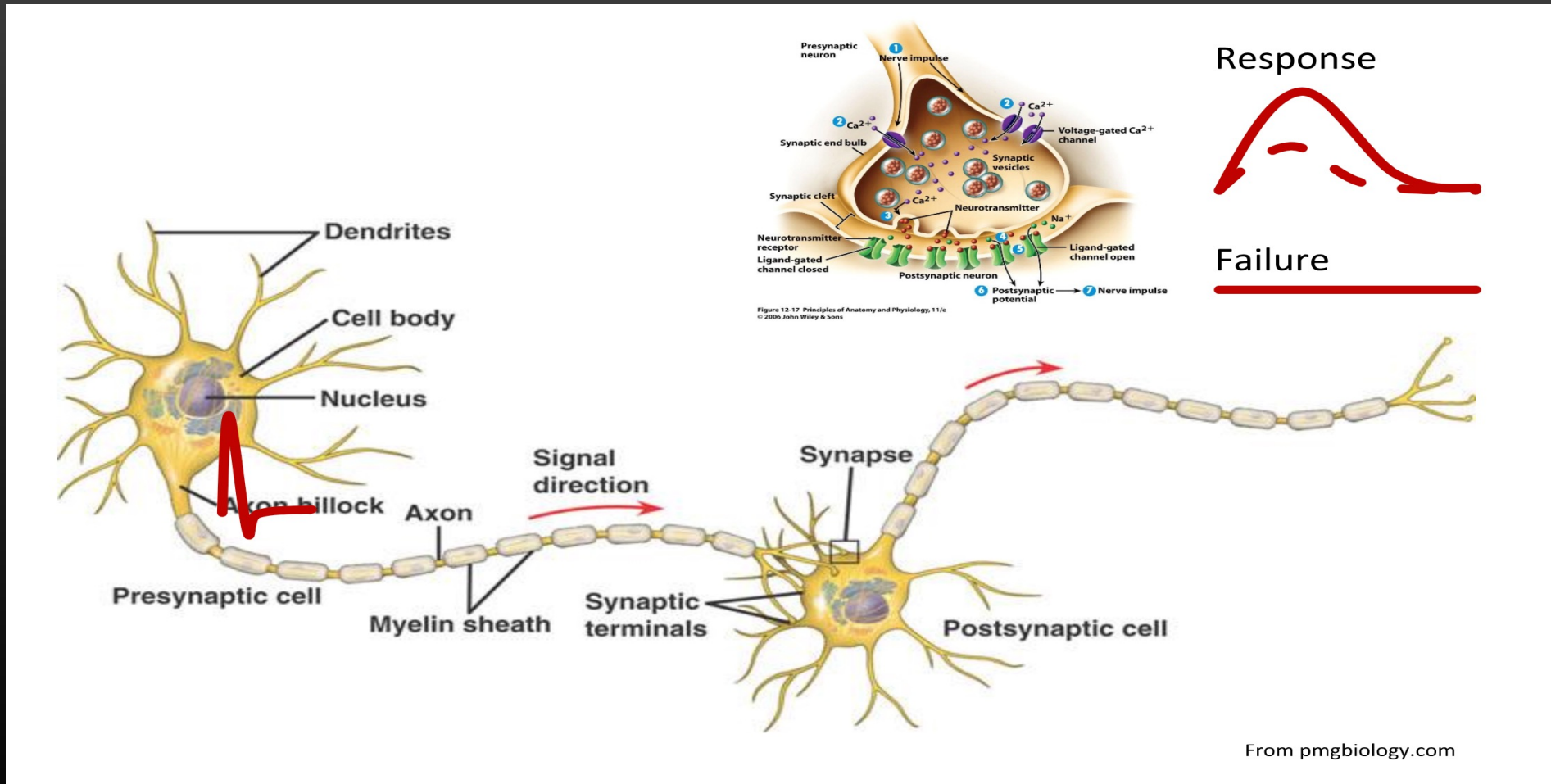
Y. Deitcher et al, Cerebral Cortex, 2017.

L. Kanari et al, Neuroinformatics, 2018.

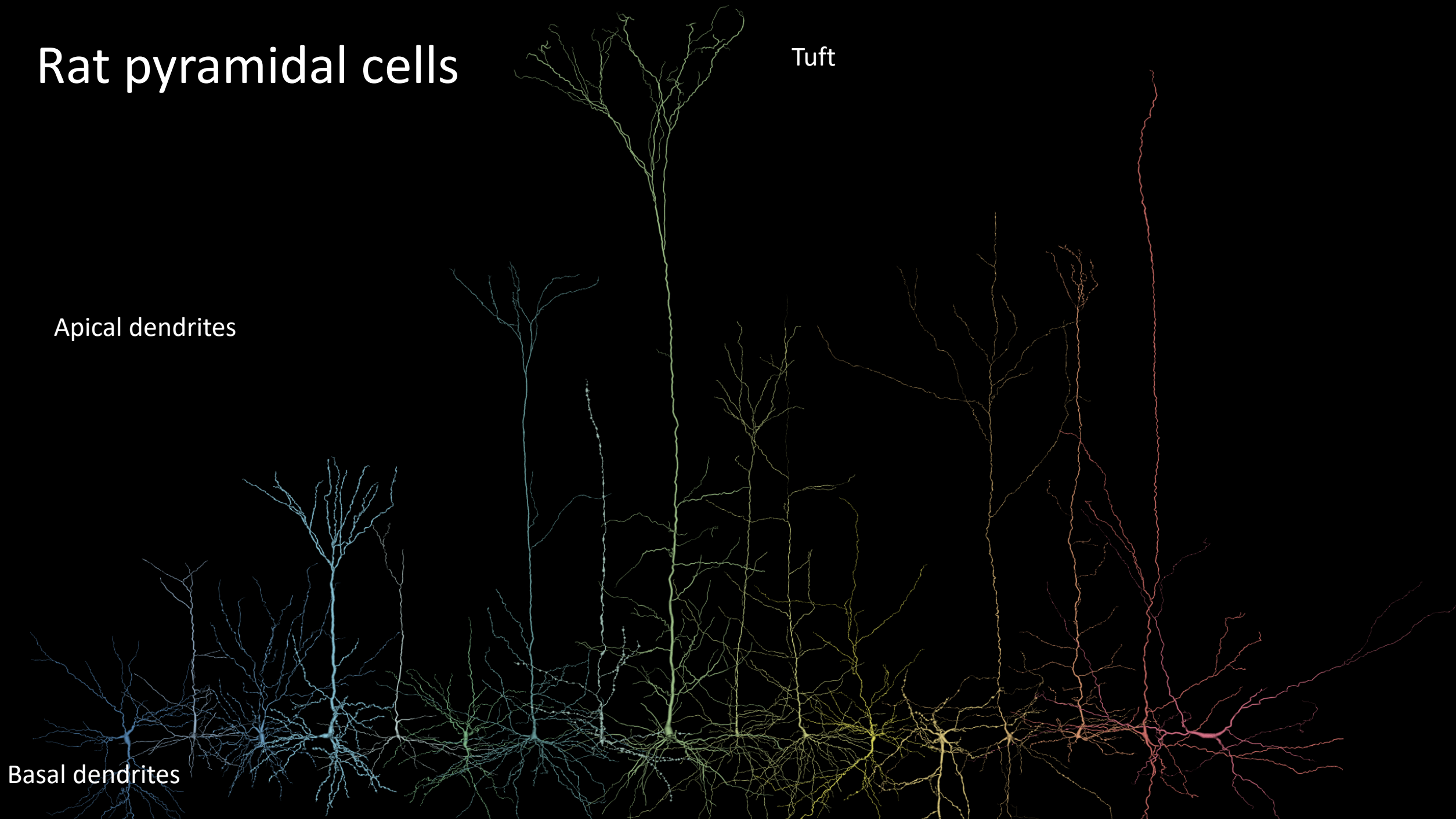
L. Kanari et al, Cerebral Cortex, 2019.

G. Colombo et al, Nature Neuroscience, 2022.

# A (very) brief intro to neurobiology



# Rat pyramidal cells

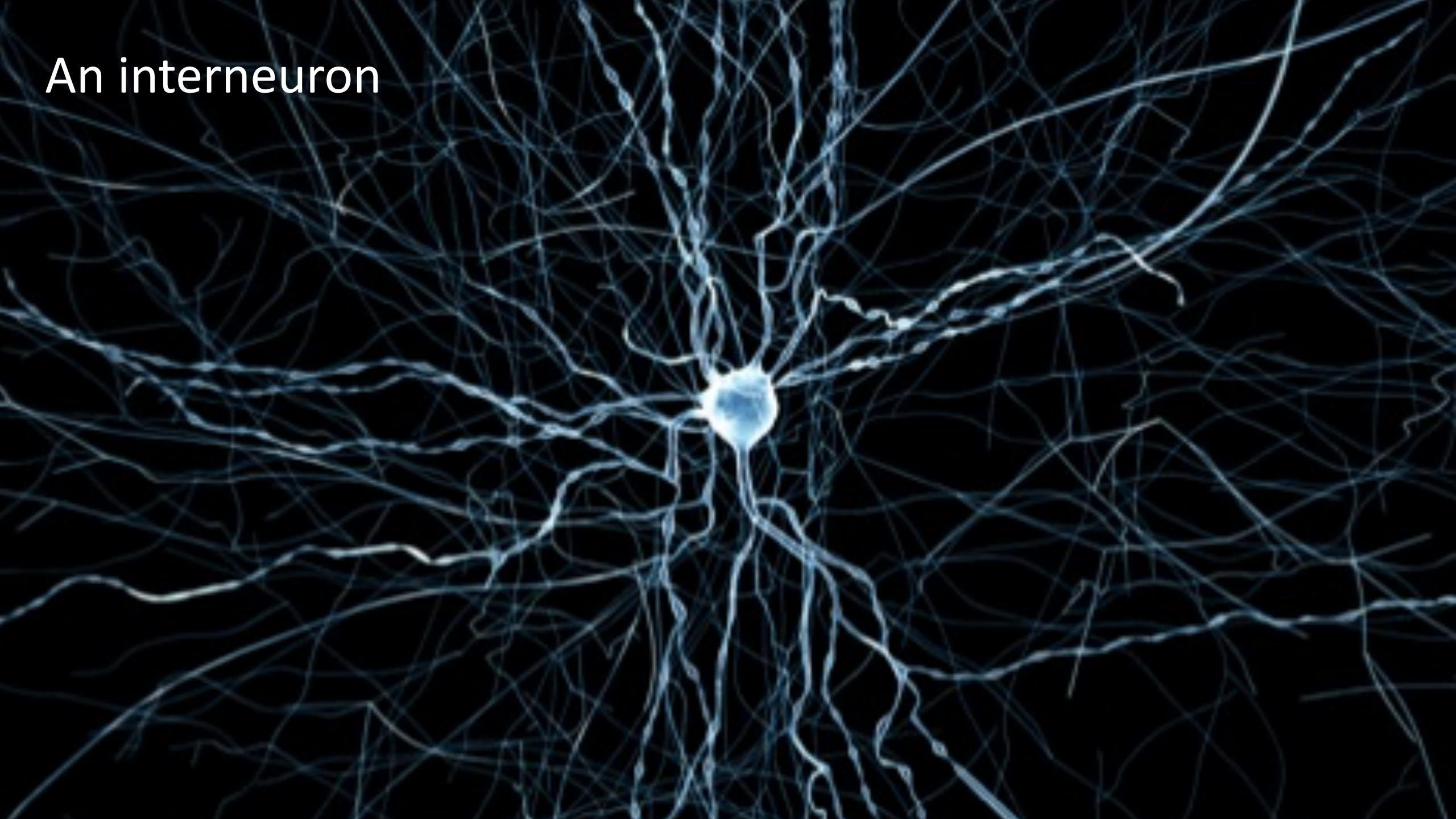


Tuft

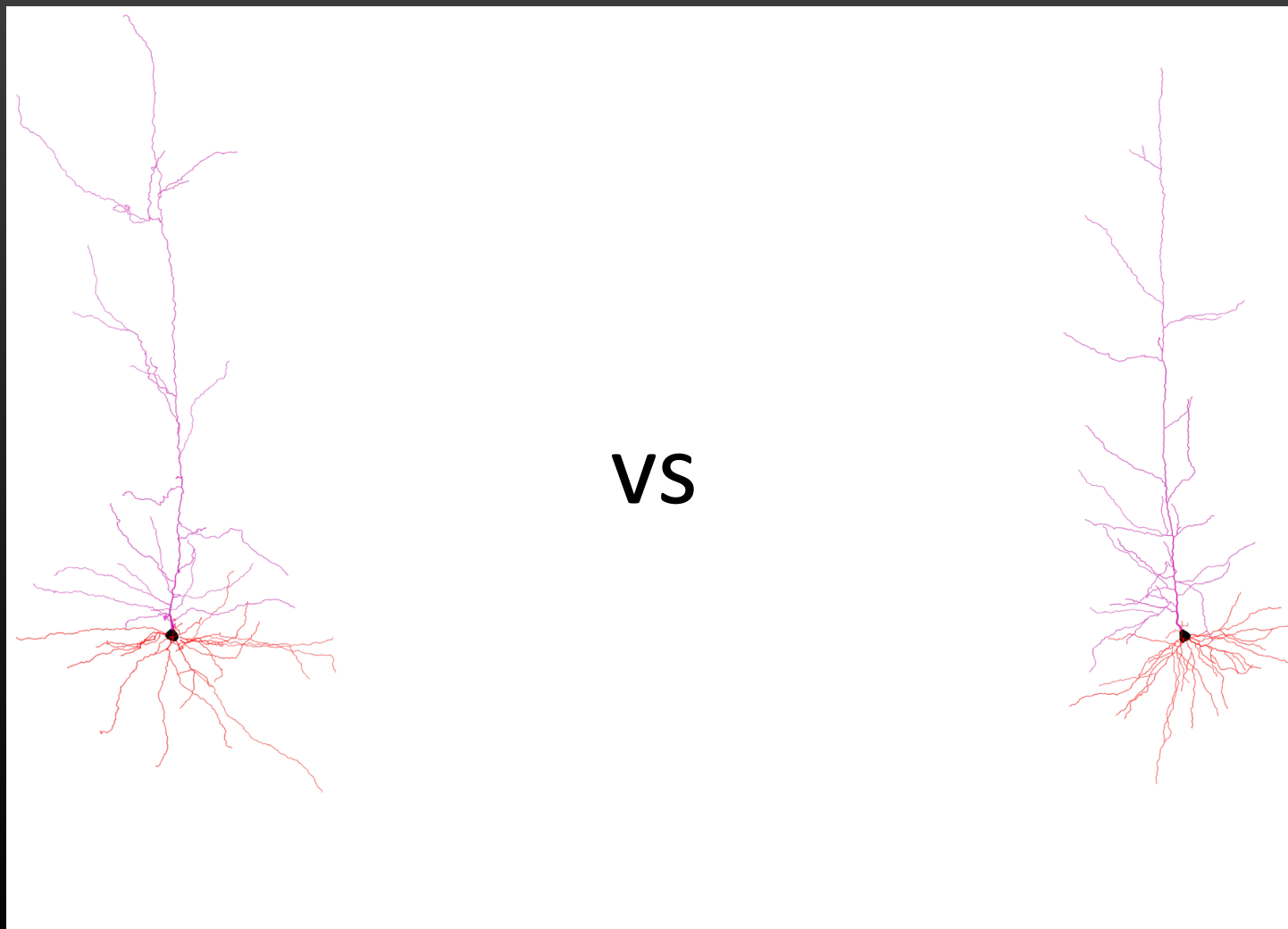
Apical dendrites

Basal dendrites

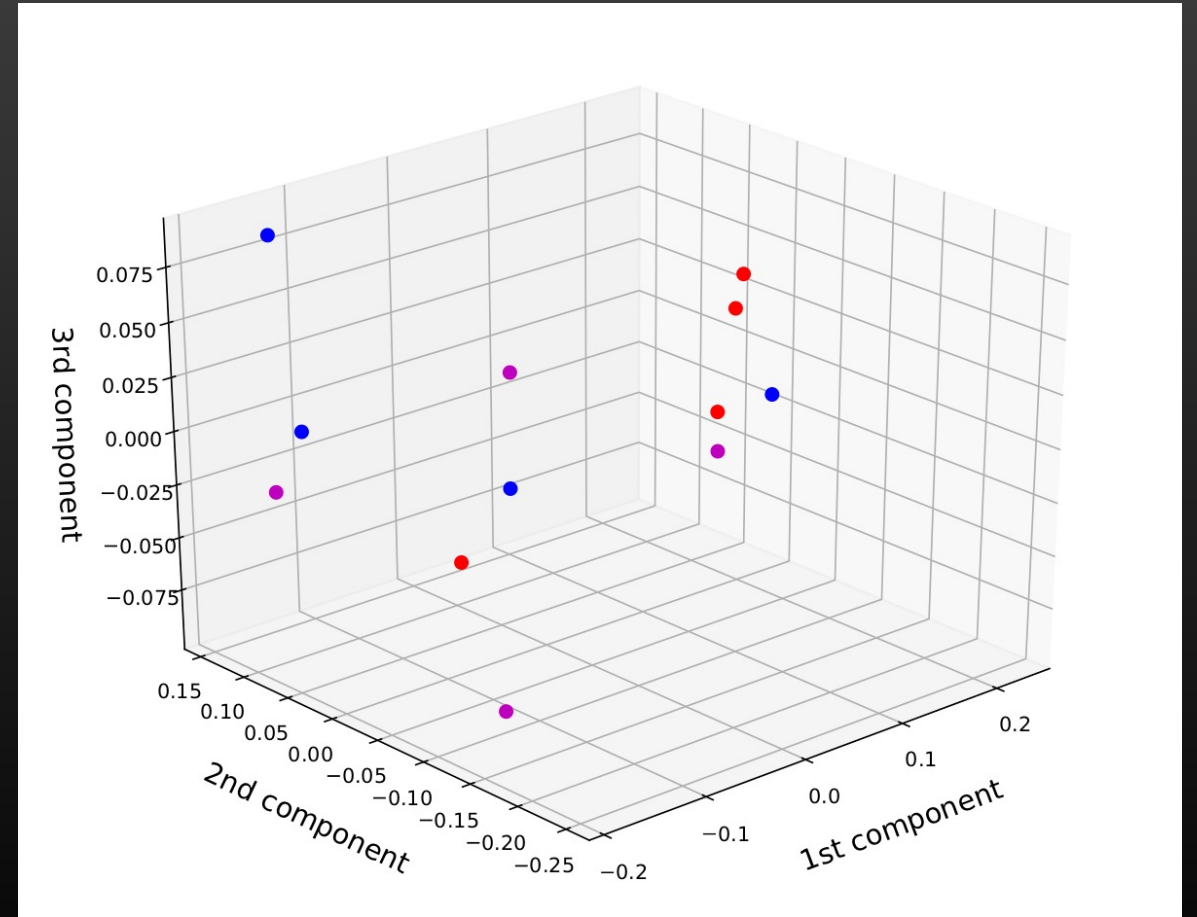
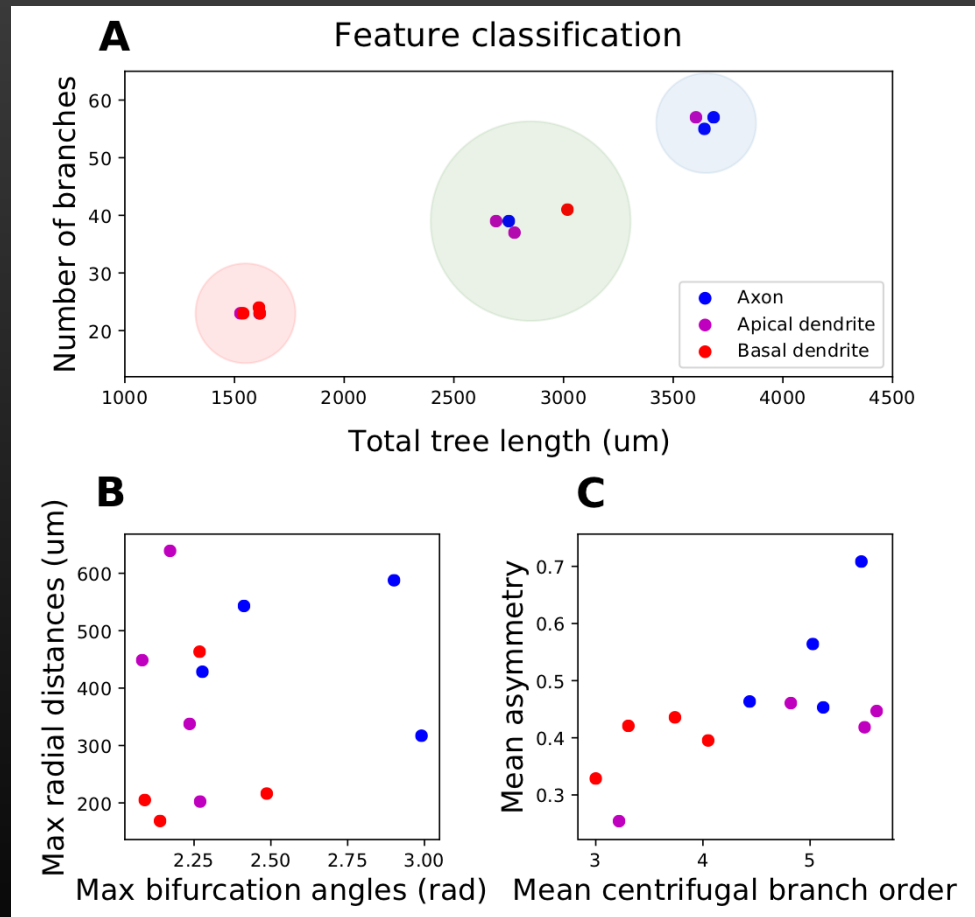
An interneuron



# How to classify neuron morphologies?



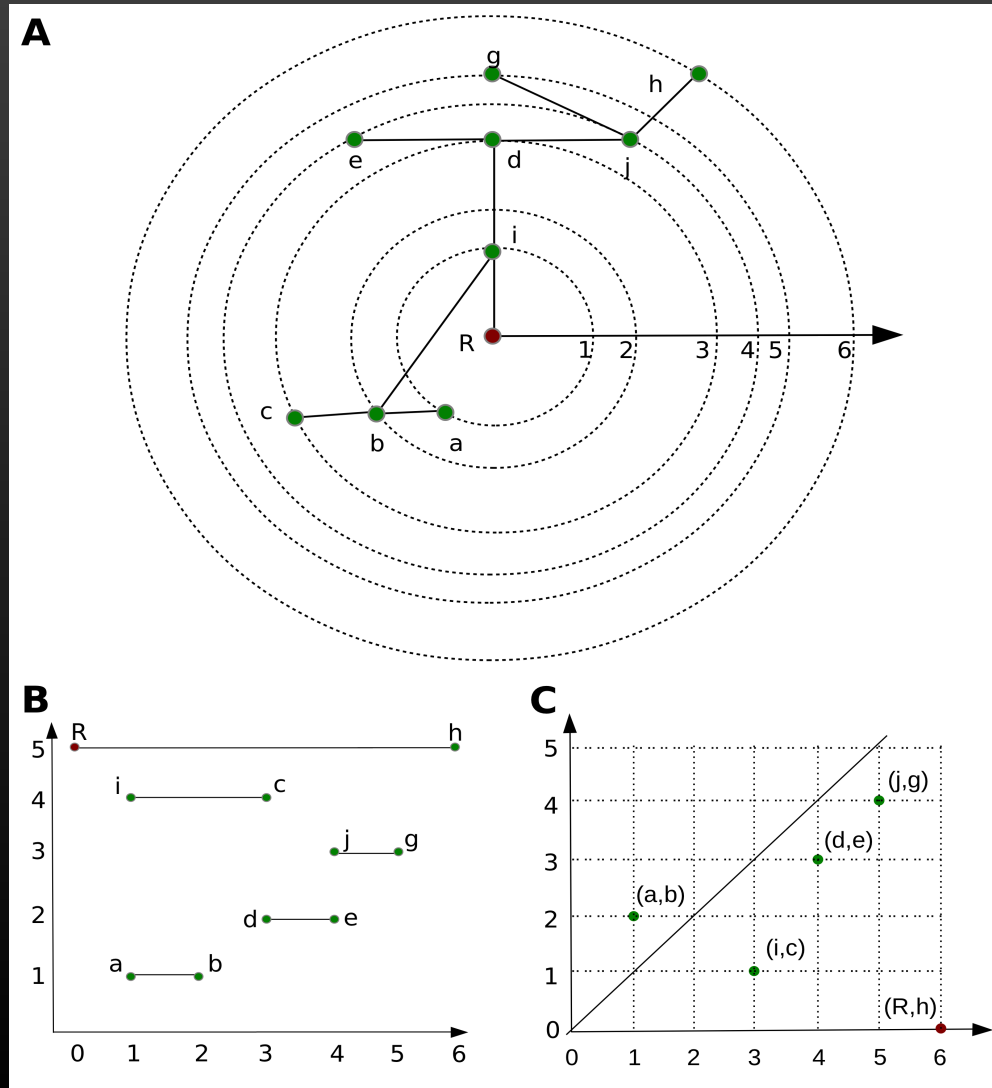
# Standard morphometrics don't suffice





i) The TMD

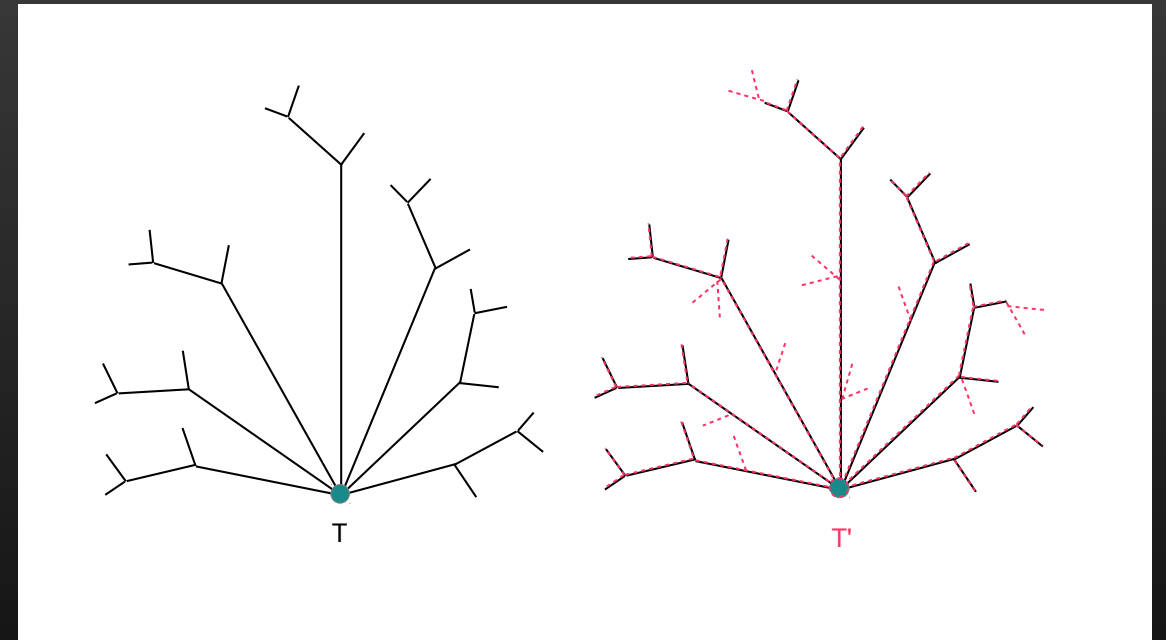
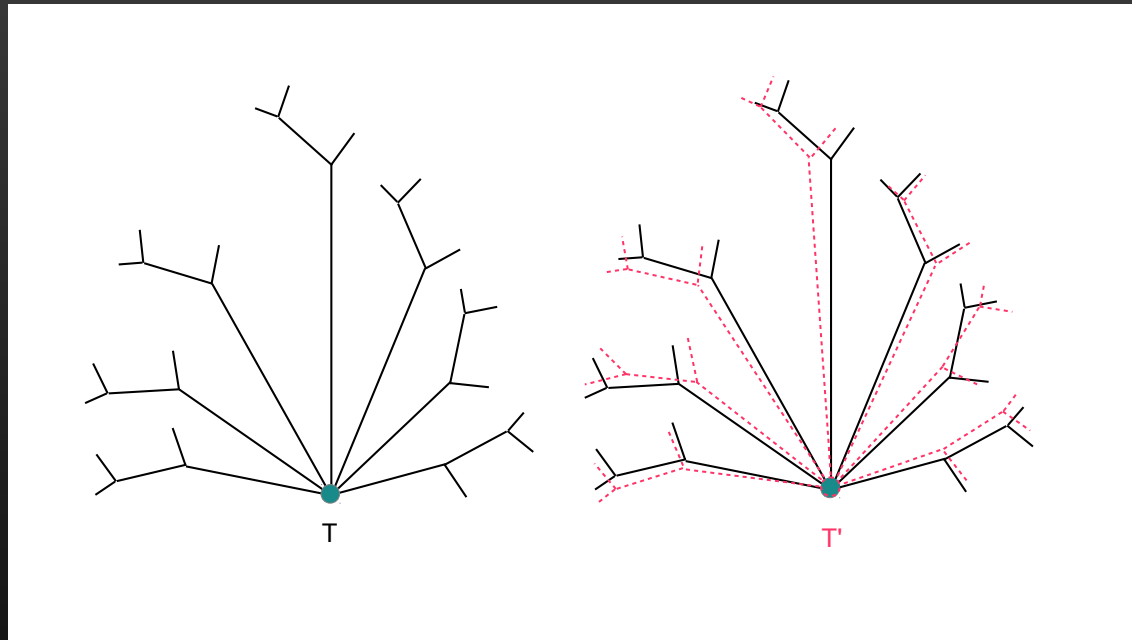
# The TMD algorithm



**Idea:** Starting at the leaves and descending recursively to the root, decompose the tree into branches, while respecting the **Elder Rule**, i.e., at any bifurcation, the elder (longer) branch survives and the younger branch is broken off.

Integrate the **topology of the tree** and the **geometry of its embedding** in space into a surprisingly powerful **global descriptor**.

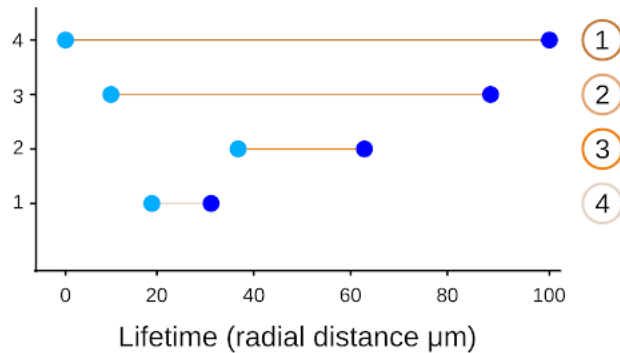
# Possible small errors and stability



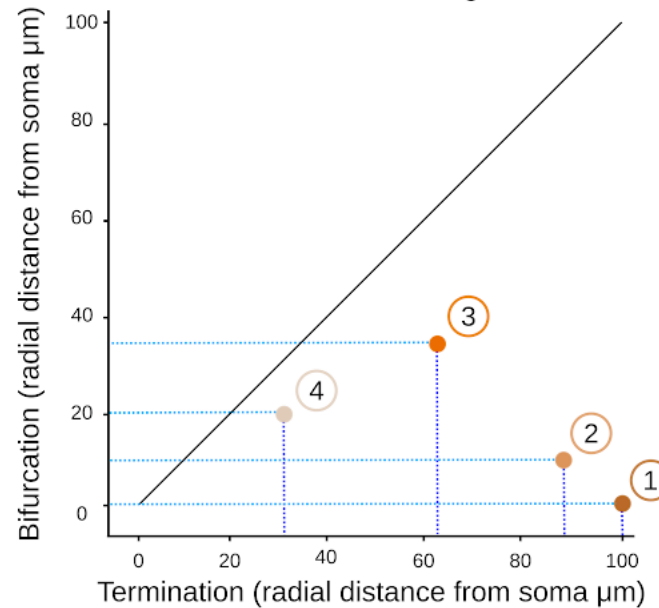
**Theorem:** The TMD is stable with respect to small errors of reconstruction, for both the bottleneck and the 1-Wasserstein distances.

# Alternative representations

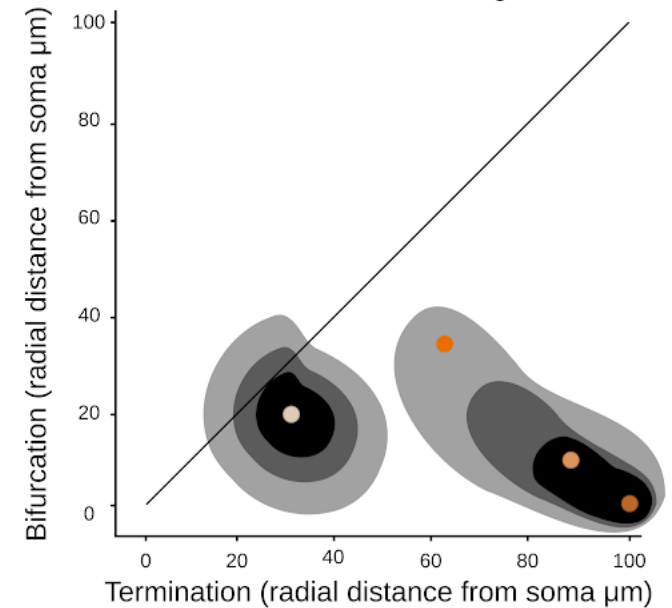
B. Persistence barcode



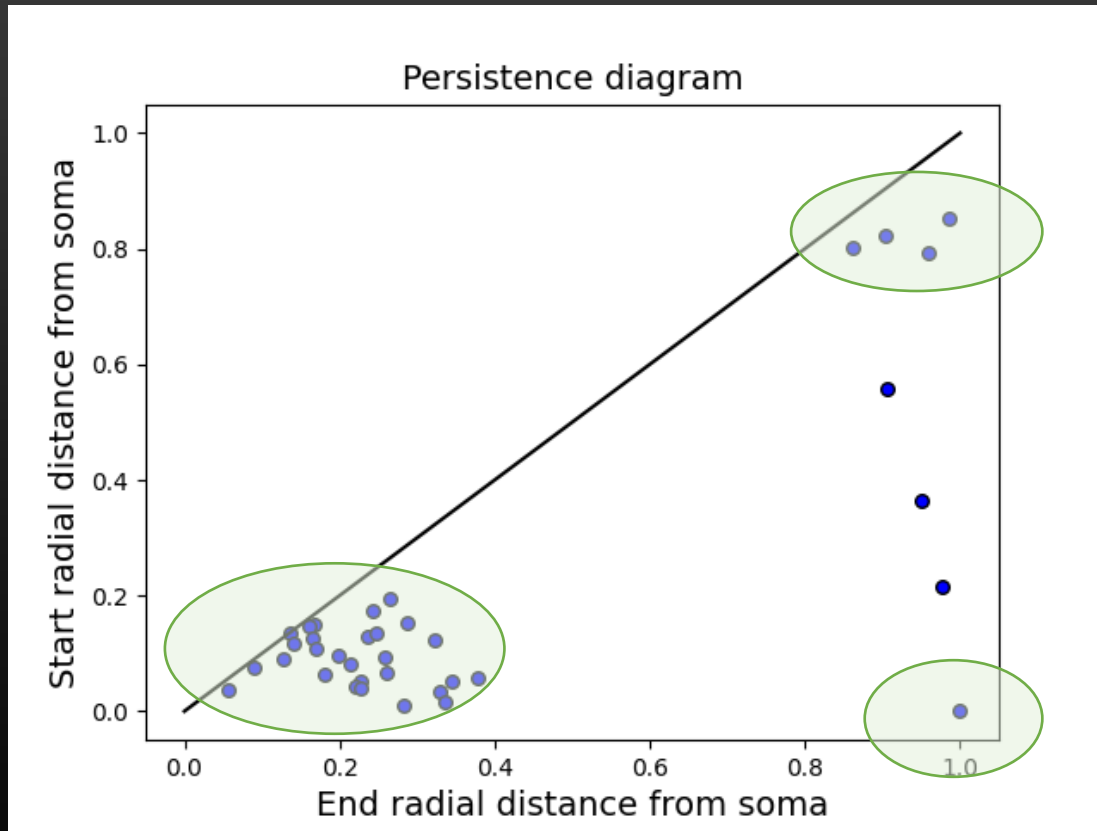
C. Persistence diagram



D. Persistence image

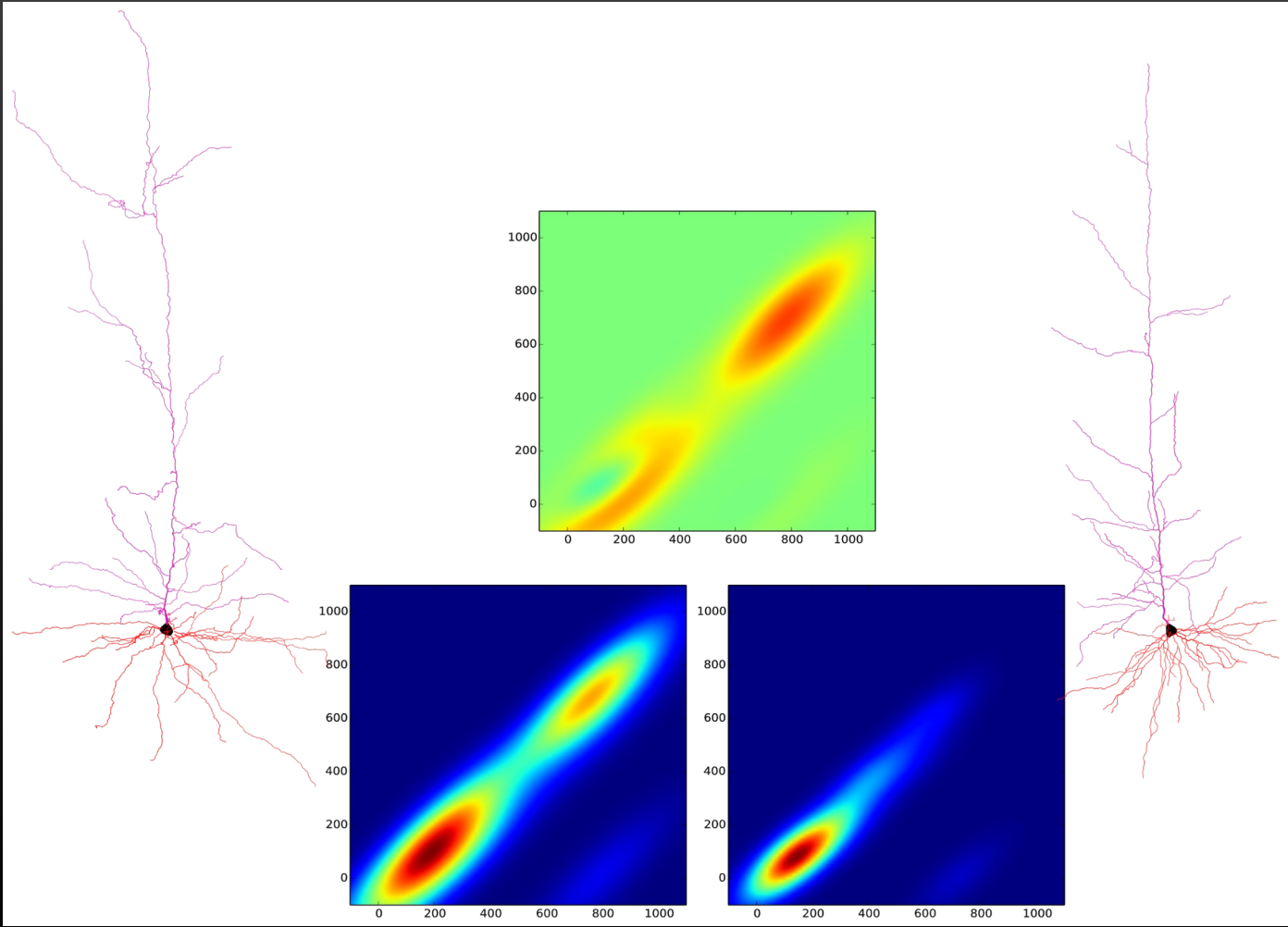


# Interpretation of the TMD



- Number of points = number of branches
- Distance from the diagonal = length of the branch
- Point farthest from the diagonal = apical main trunk
- Points near the origin = obliques near the soma
- Points far from the origin, near the diagonal = apical tuft far from the soma

# The TMD in our motivating example



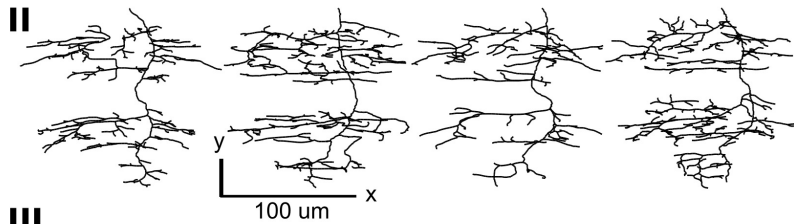
ii) Applications of the TMD

# Interspecies comparison

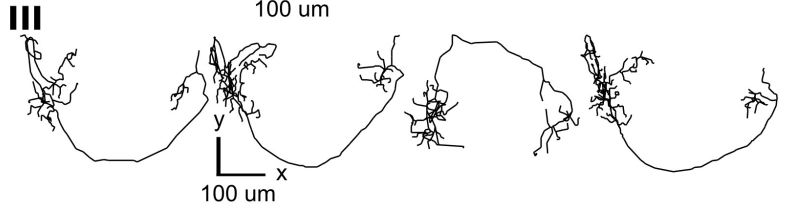
Cat



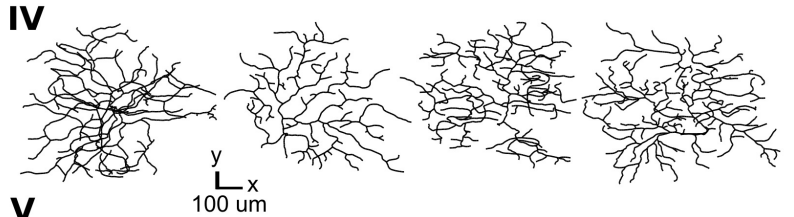
Dragonfly



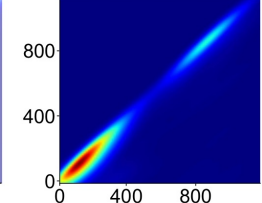
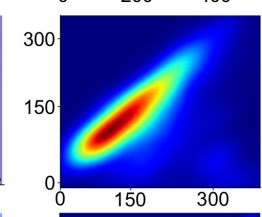
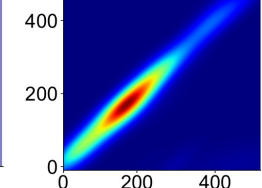
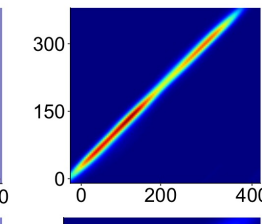
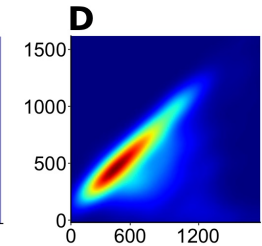
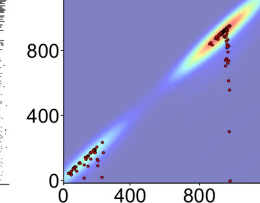
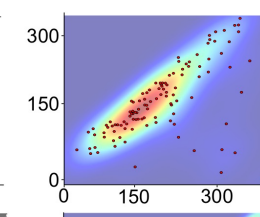
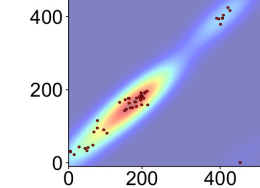
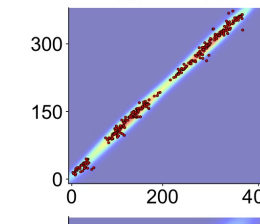
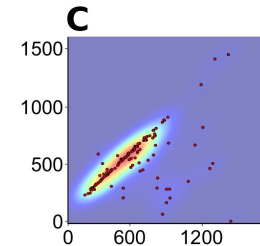
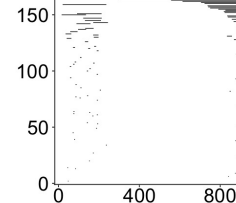
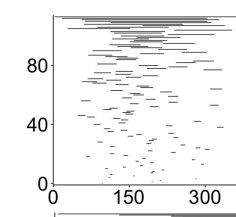
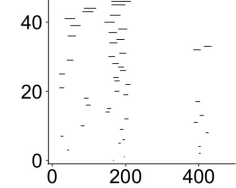
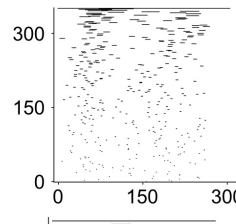
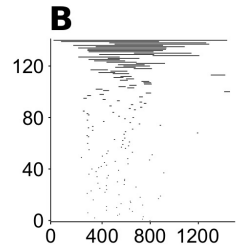
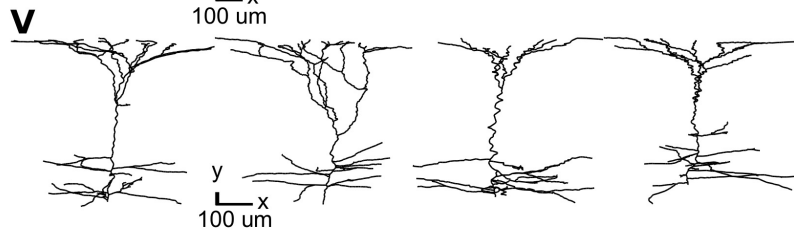
Fruitfly



Mouse

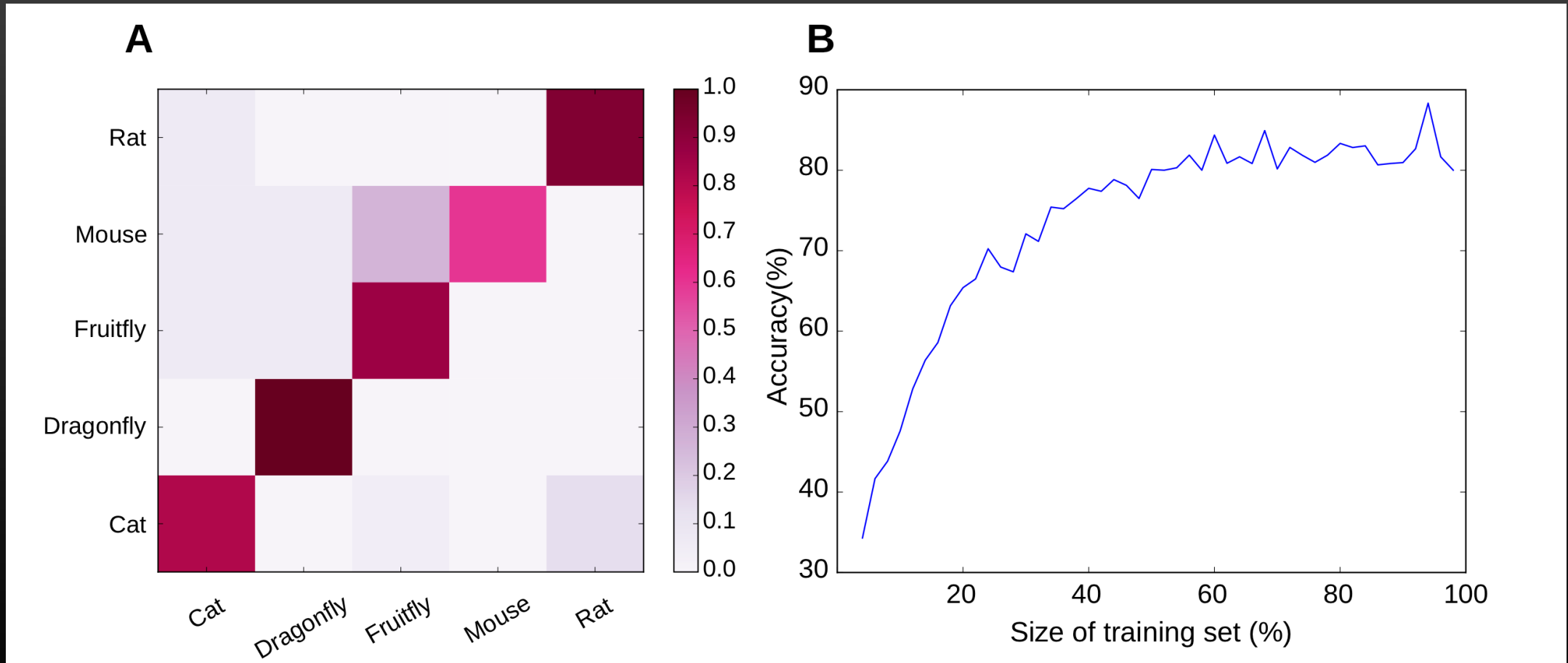


Rat

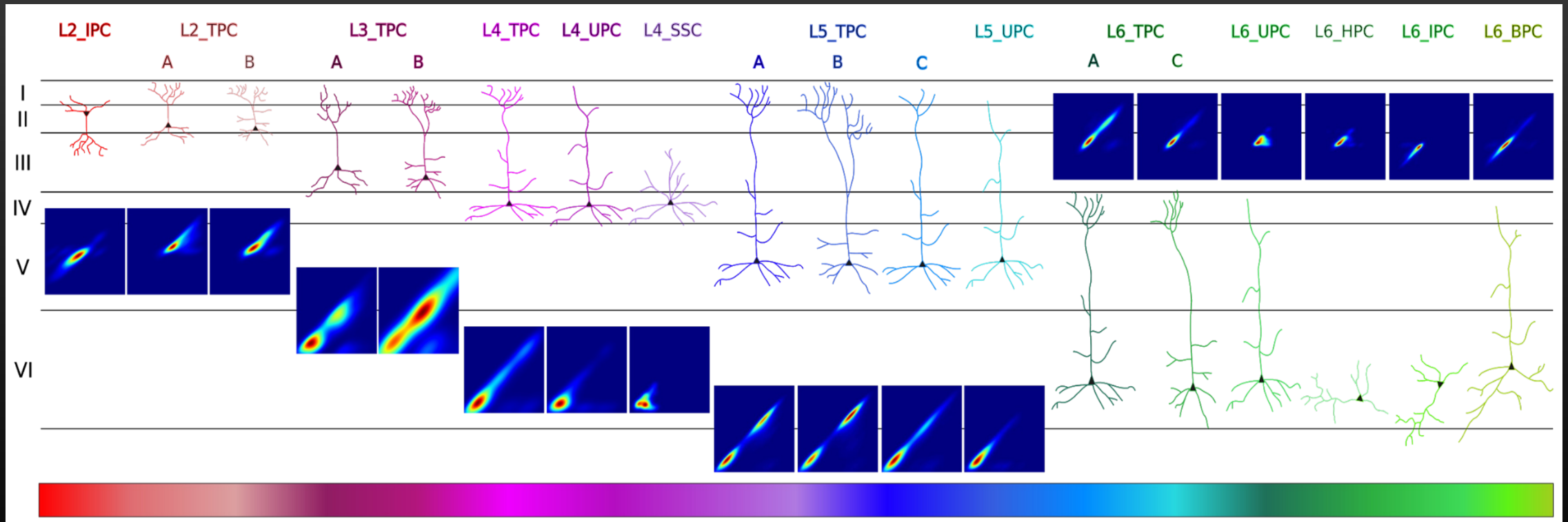




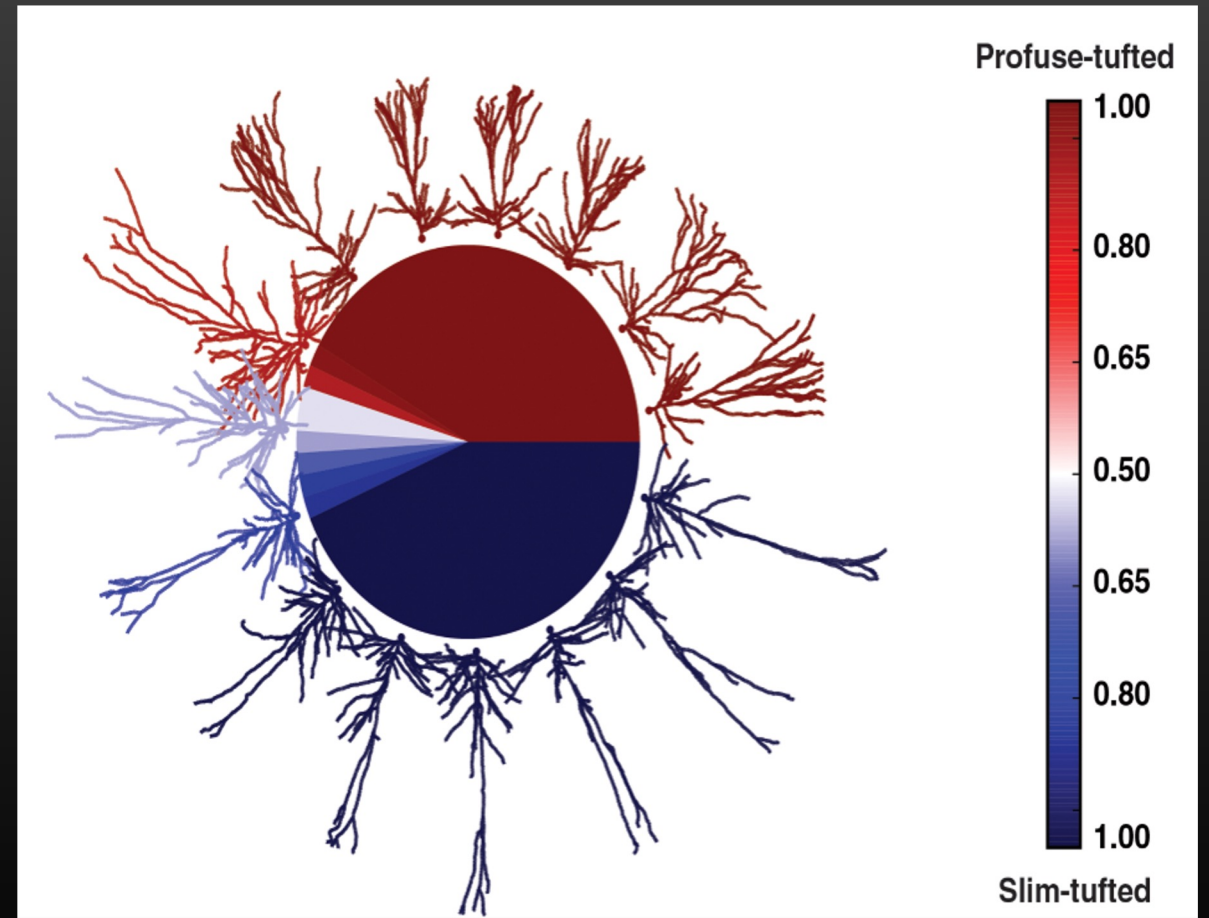
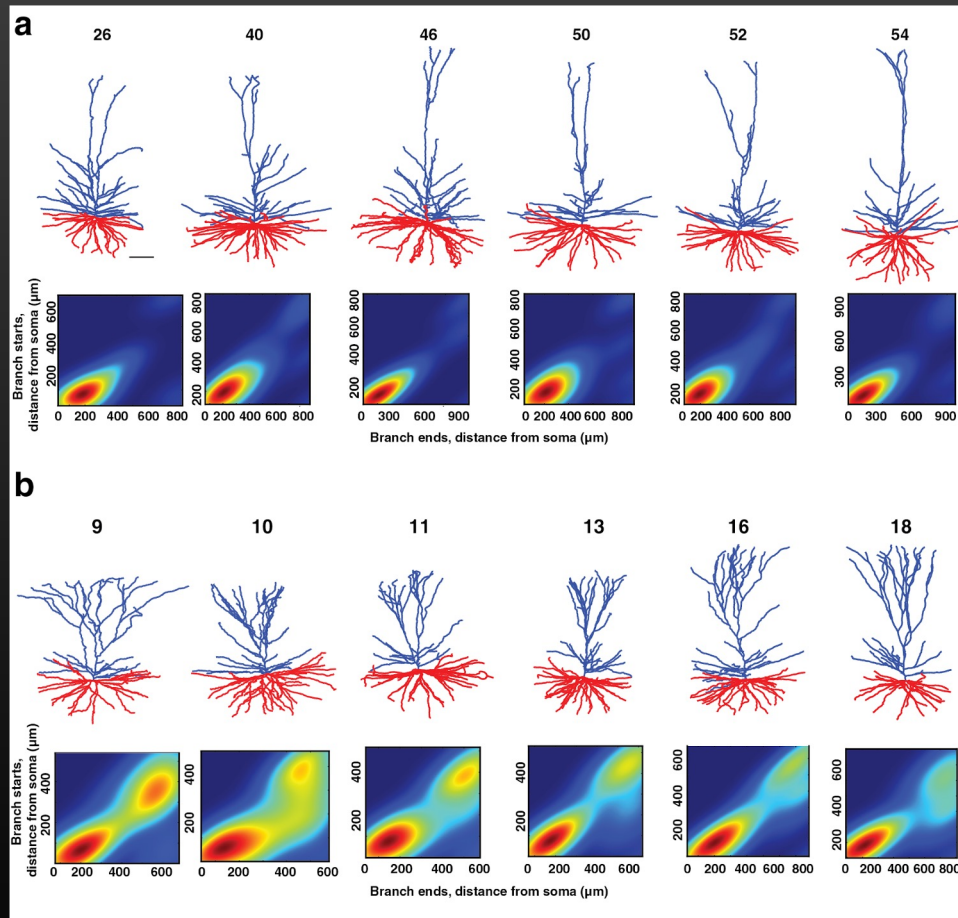
# Training the classifier



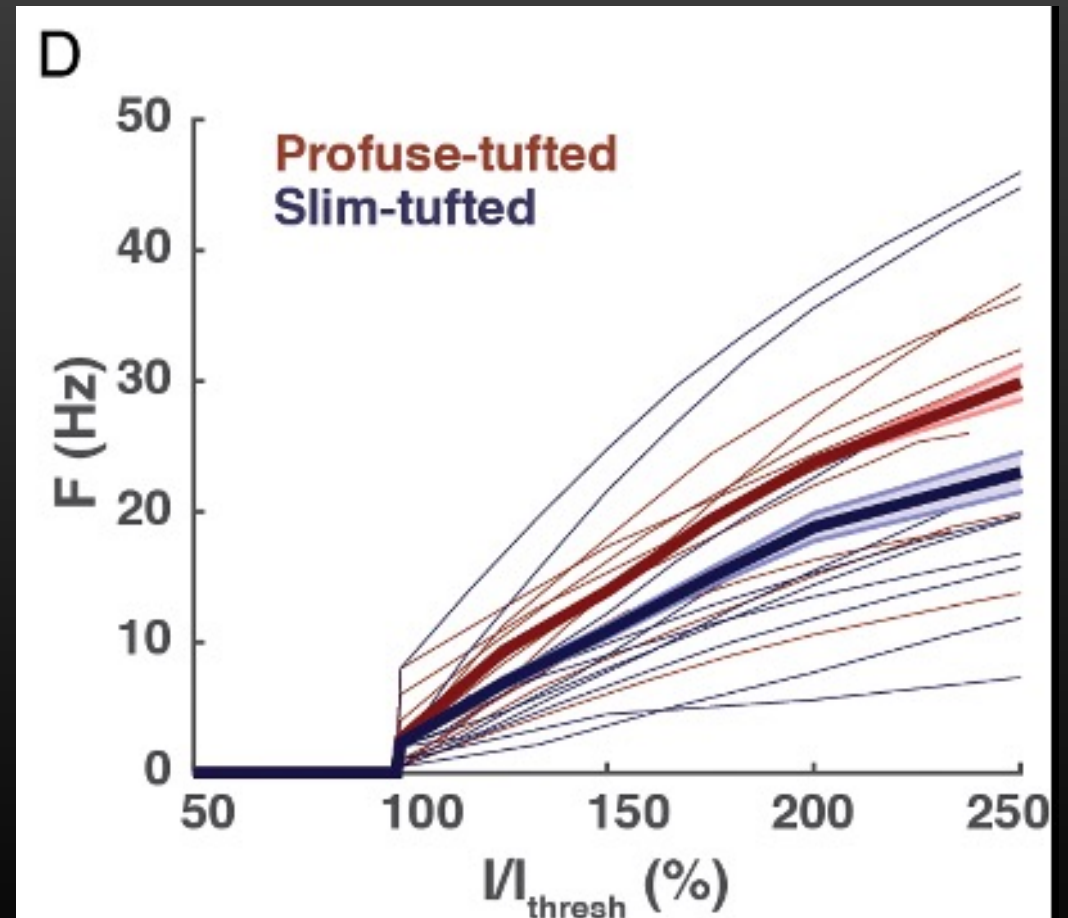
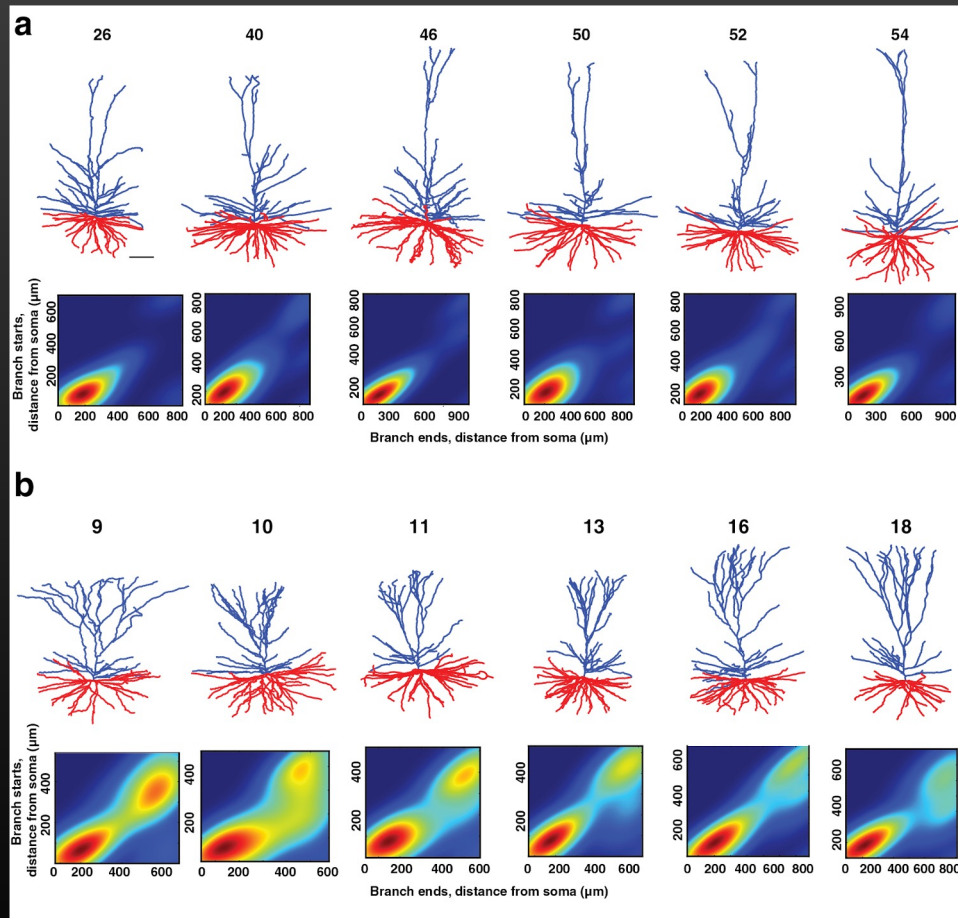
# TMD of rat pyramidal cells



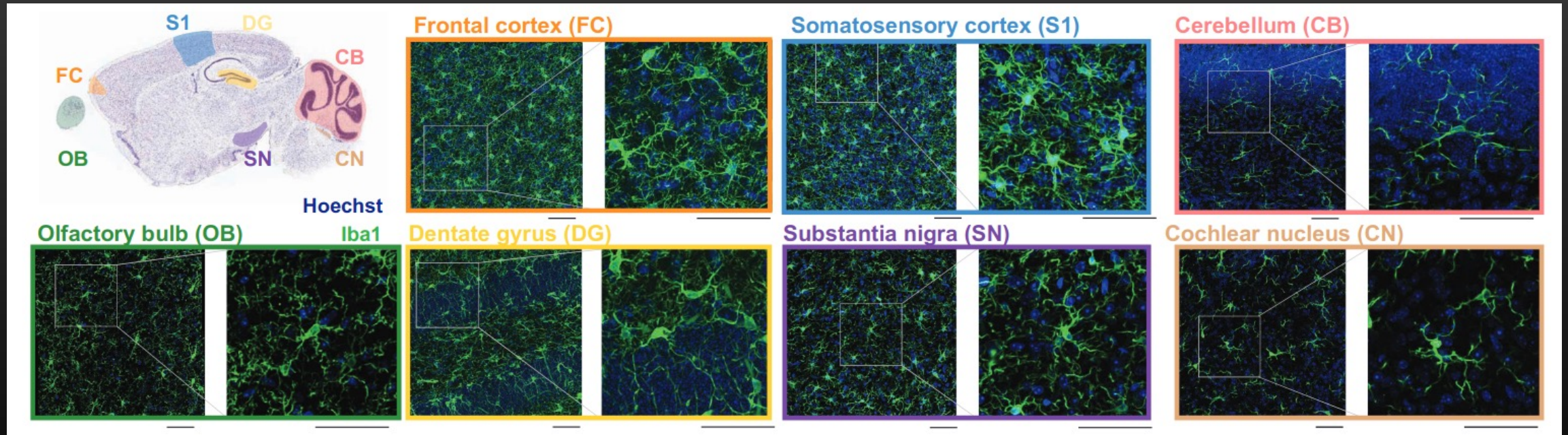
# Clustering of human pyramidal cells



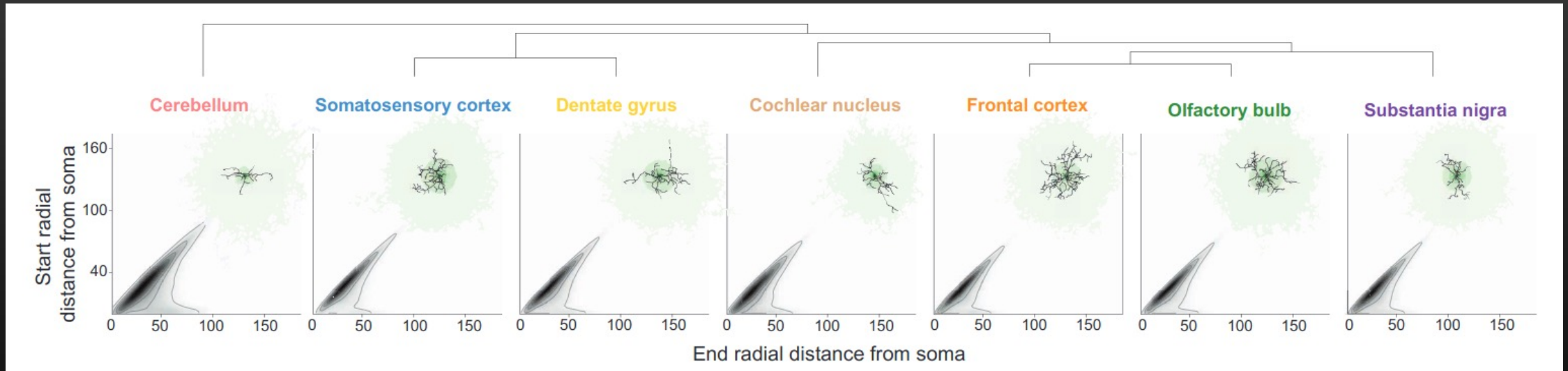
# Clustering of human pyramidal cells



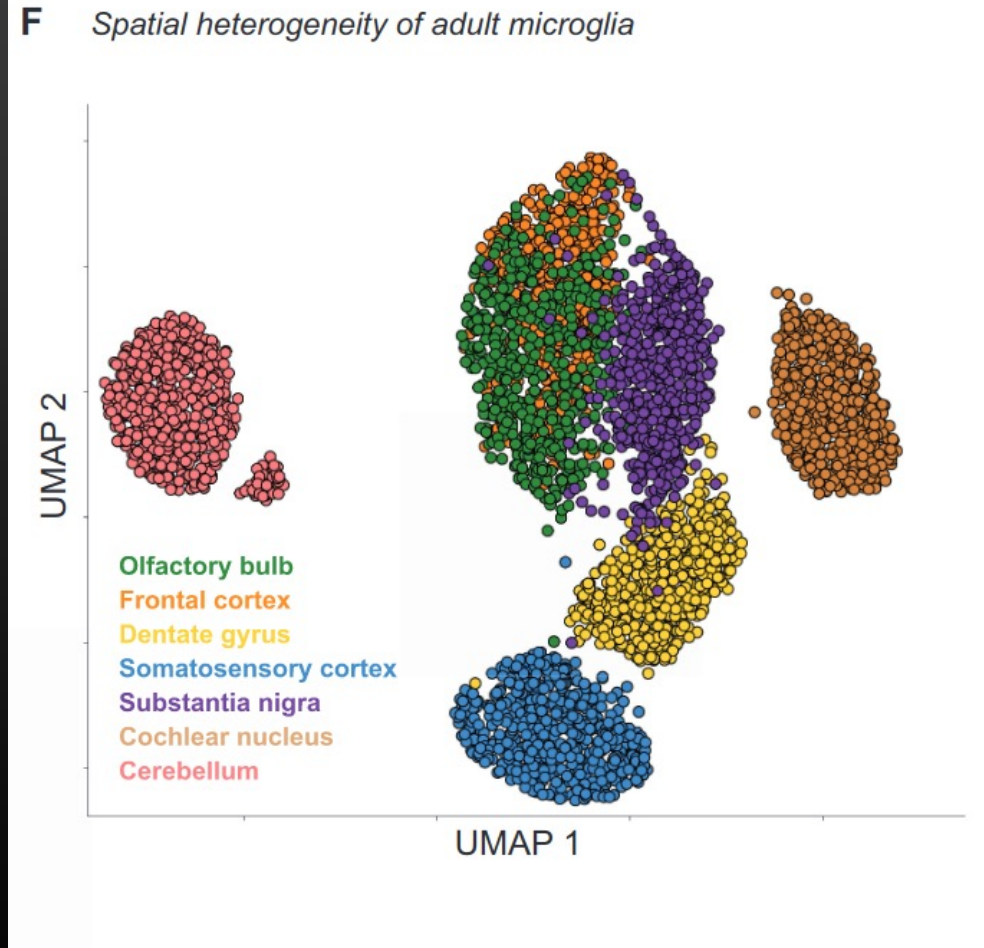
# Morphological characterization of microglia



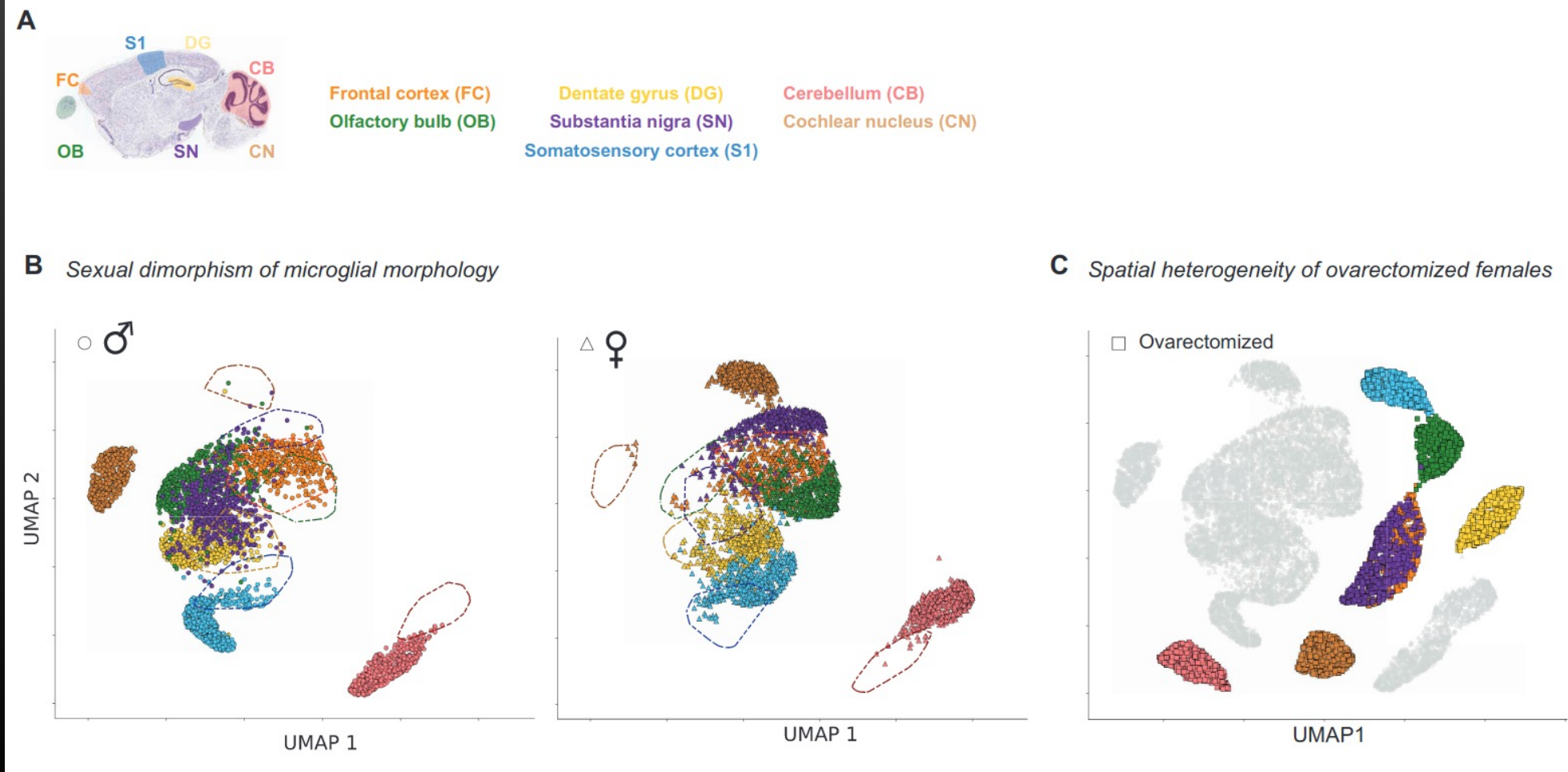
# Morphological characterization of microglia



# Morphological characterization of microglia



# Morphological characterization of microglia





# Of mice and men

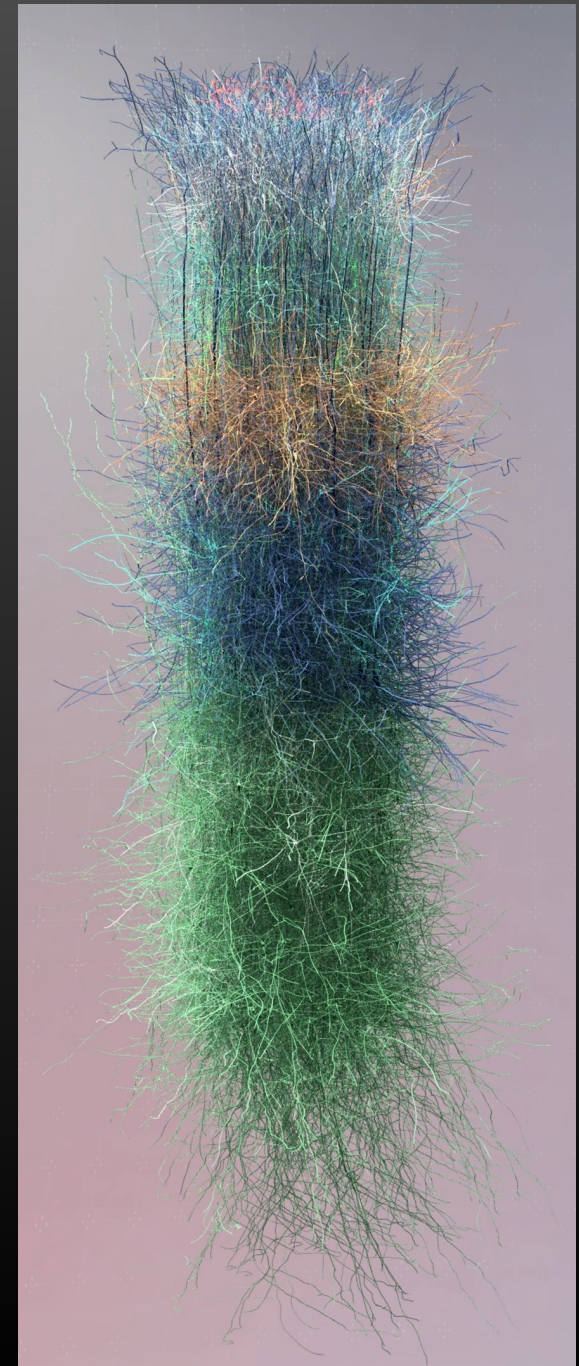
L. Kanari, et al., bioRxiv, 2023.

# Questions

- What distinguishes human from mouse neurons?
- Are the differences merely a matter of scale?
- What are the consequences of these differences for the structure and function of human and mouse connectomes?

# A connectome core sample

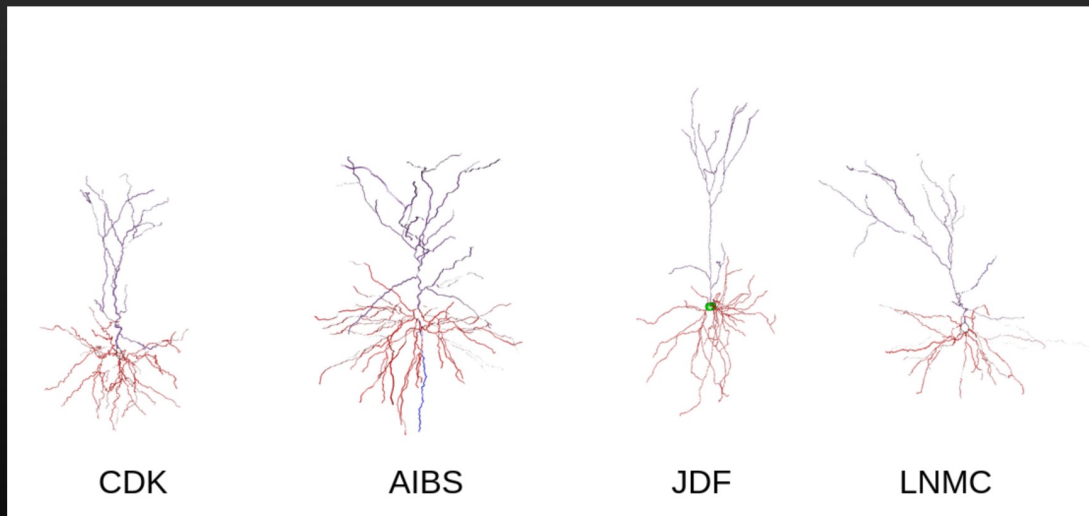
In the cortex:  
six structurally and functionally distinct layers



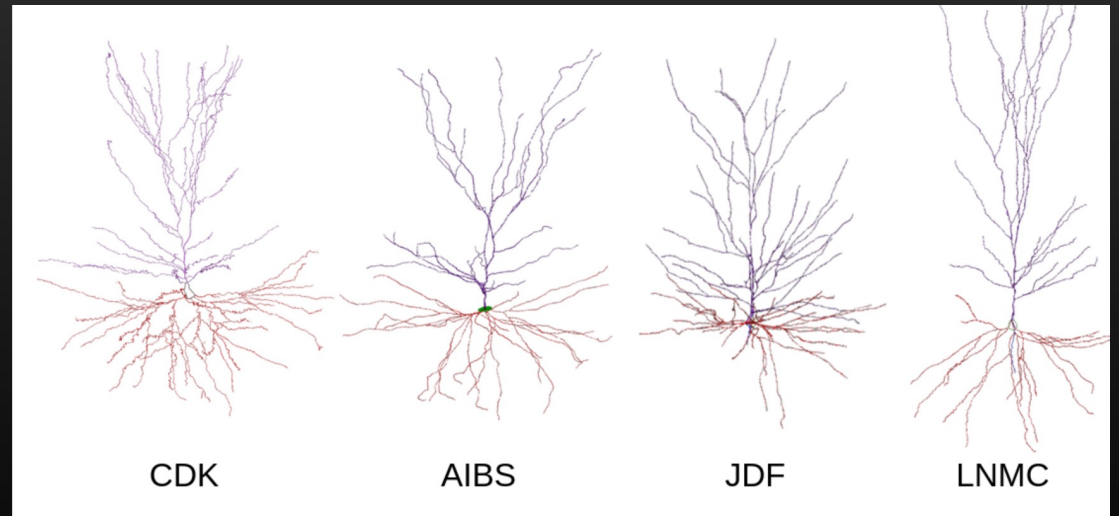
i) Comparison of neurons

# Visual comparison

## Mouse

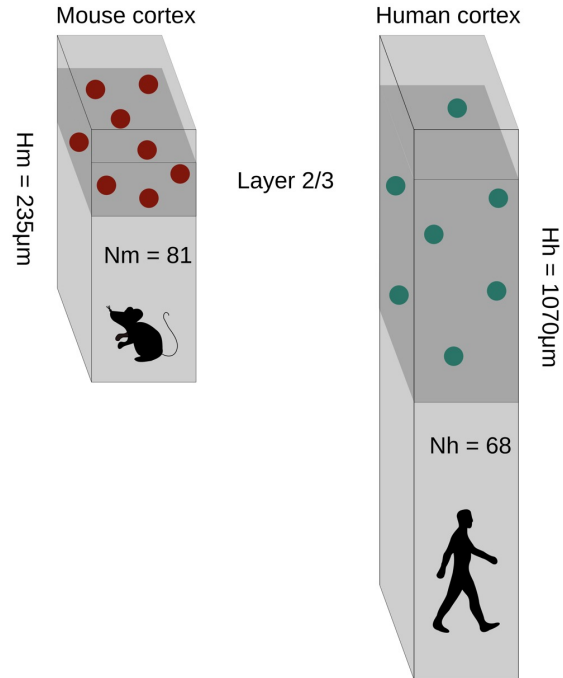


## Human

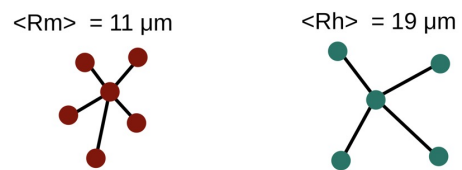


# Anatomical comparison

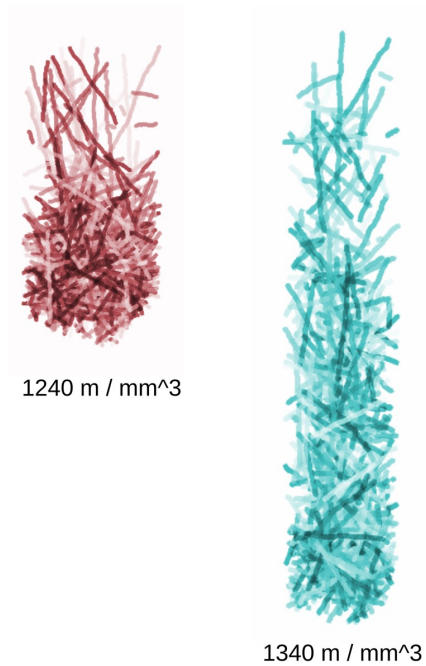
A. Neuron density decreases in human cortex



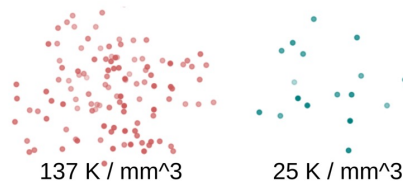
B. Average distance doubles in human cortex



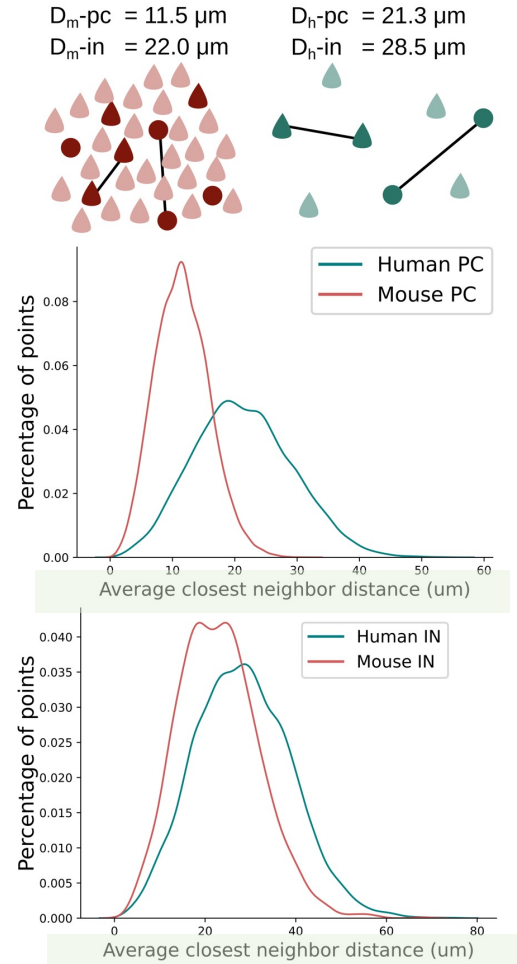
C. Dendritic density preserved within cortical layers



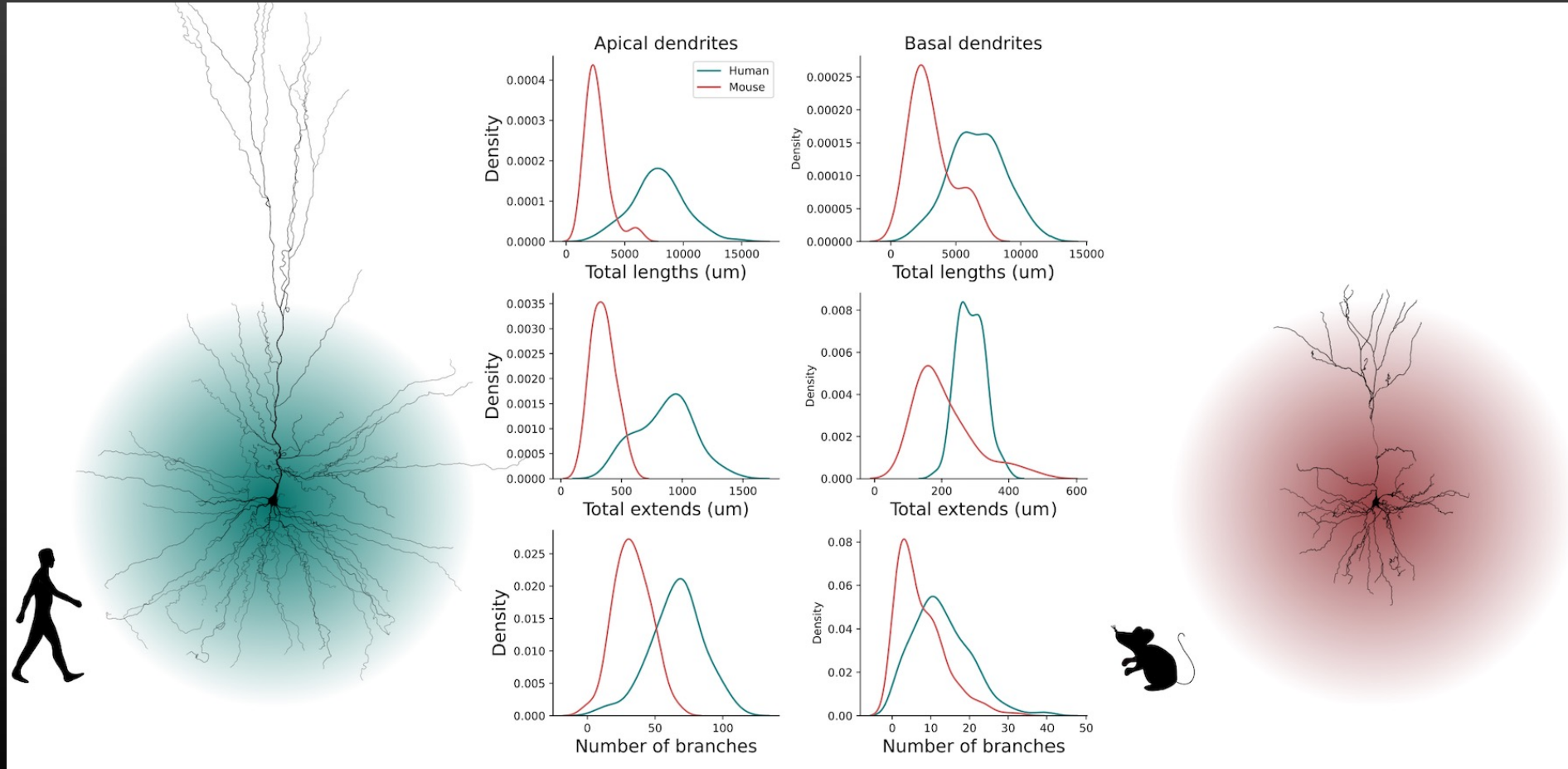
D. Layer 2/3 neurons are sparser in human



E. Non-uniform scaling of distances for PC - IN

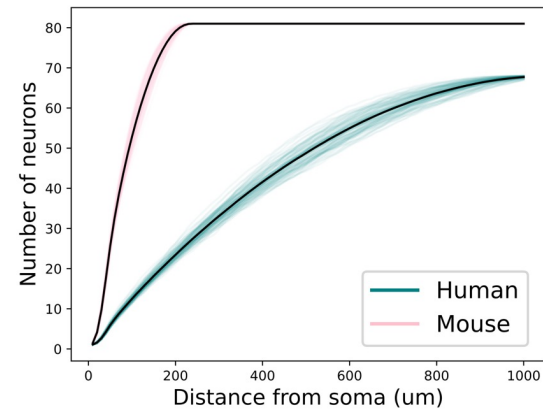


# Anatomical comparison

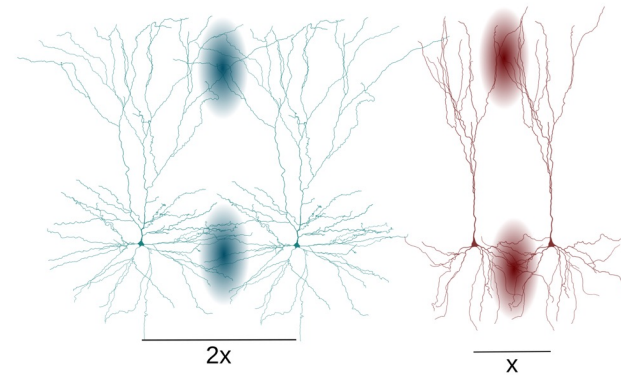


# Anatomy and connectivity

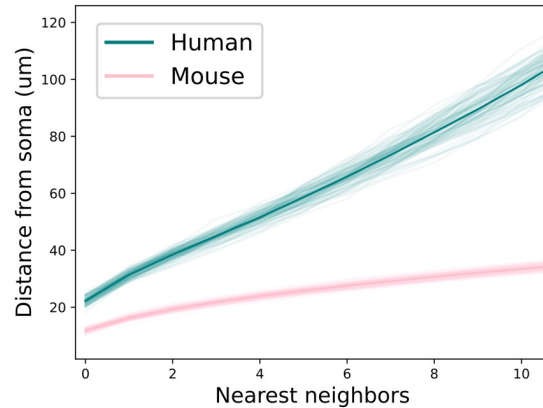
A. Distribution of neurons within a cortex slice



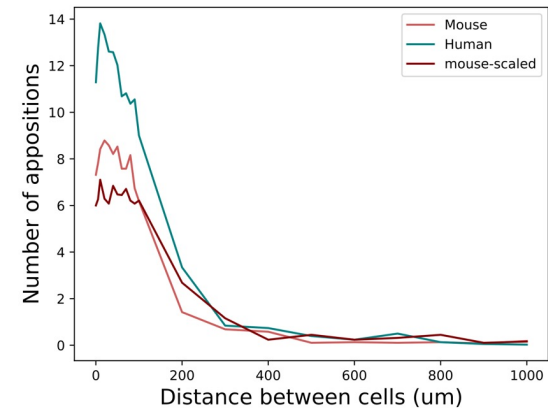
C. Increased intra-neuron distances changes connection probability



B. Distribution of distances around soma

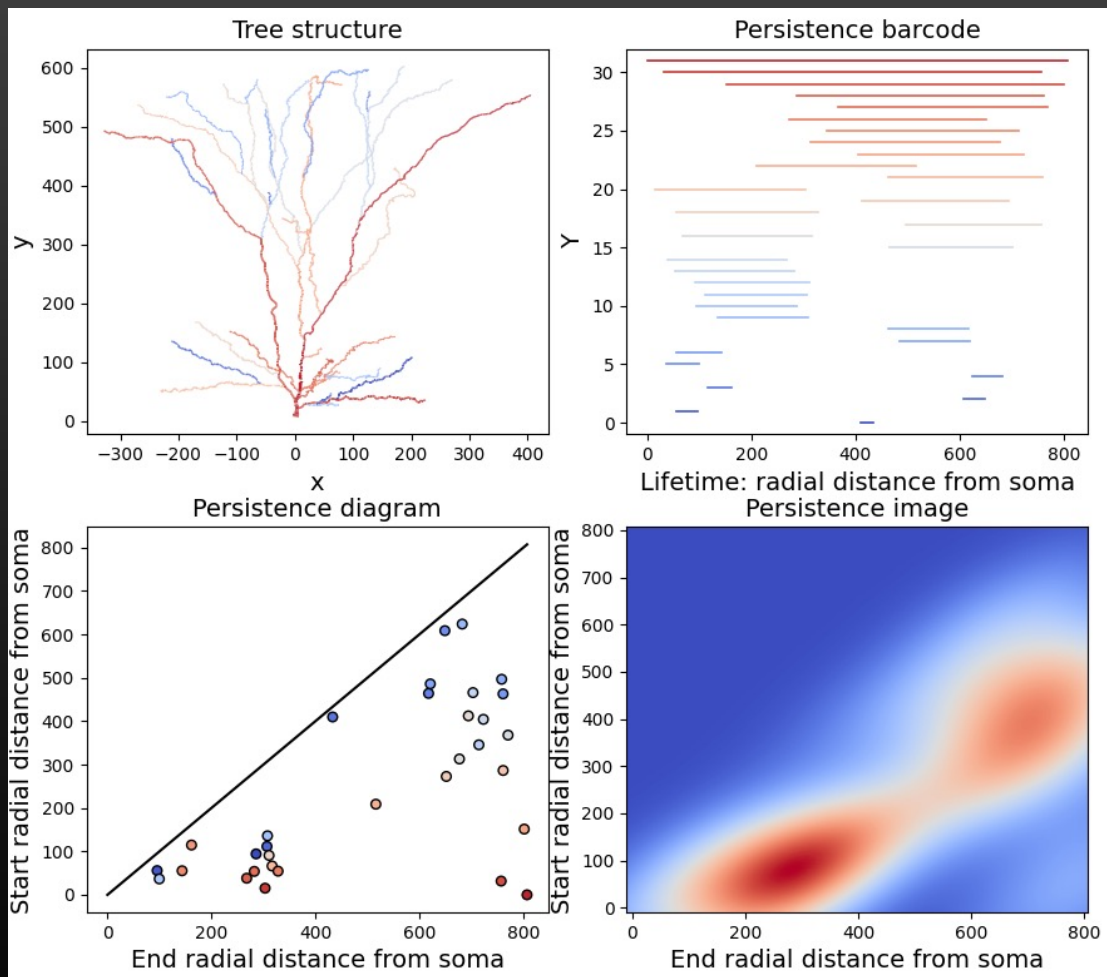


D. Size does not preserve connection probability

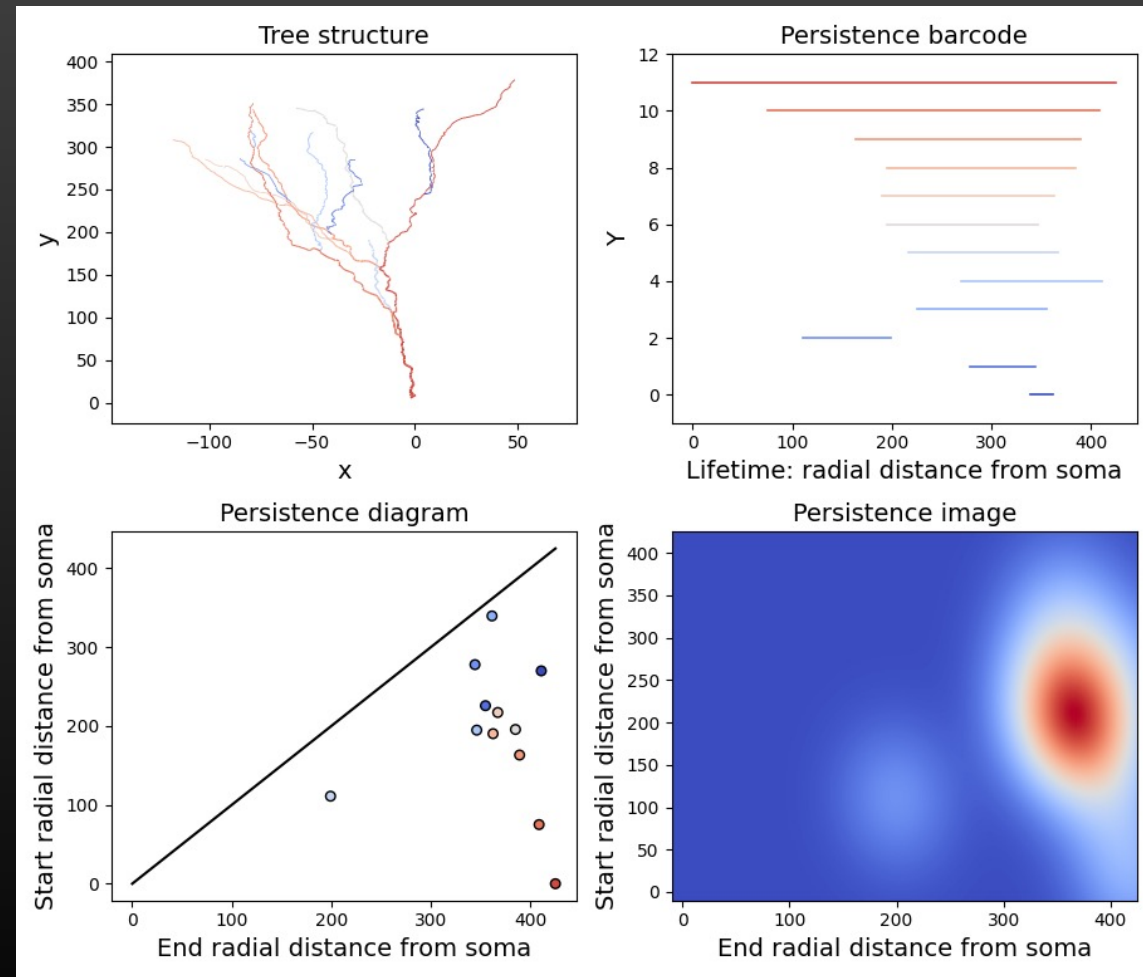




# Topological comparison



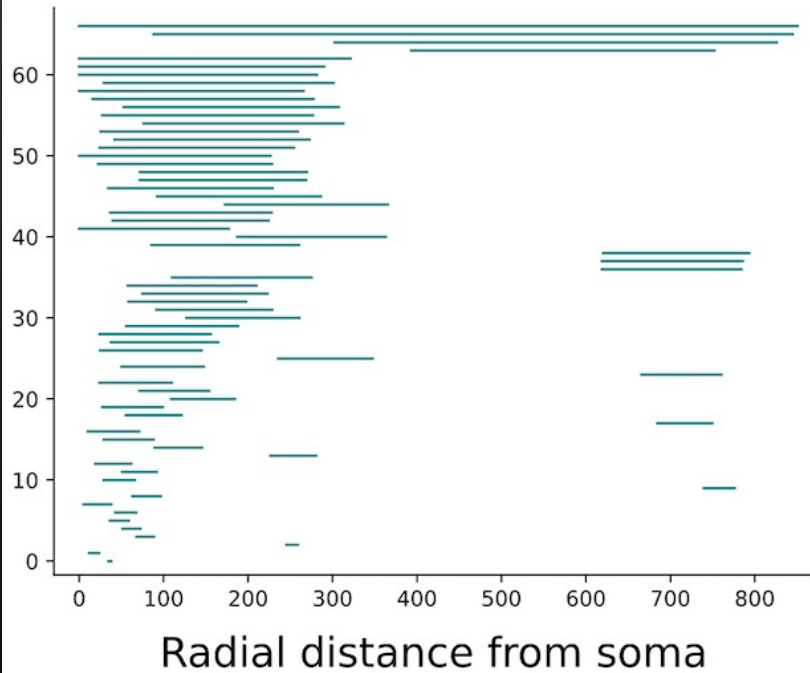
Human neuron



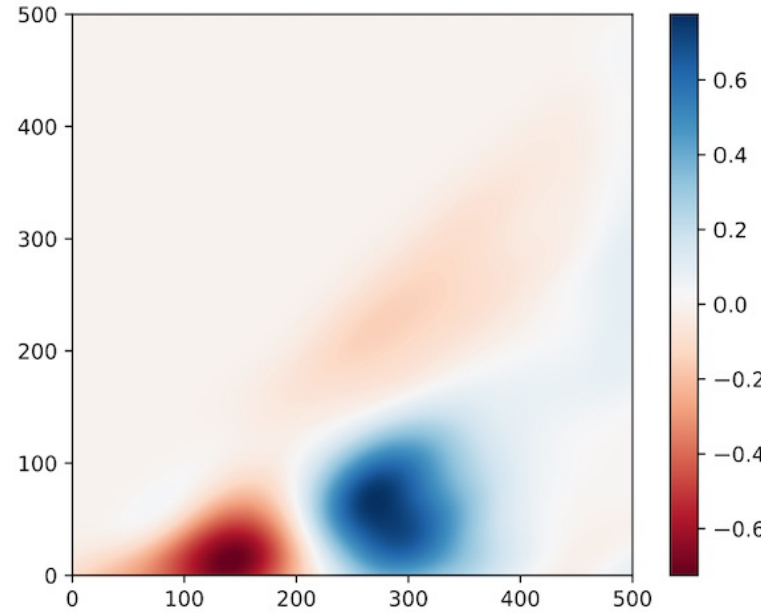
Mouse neuron

# Topological comparison

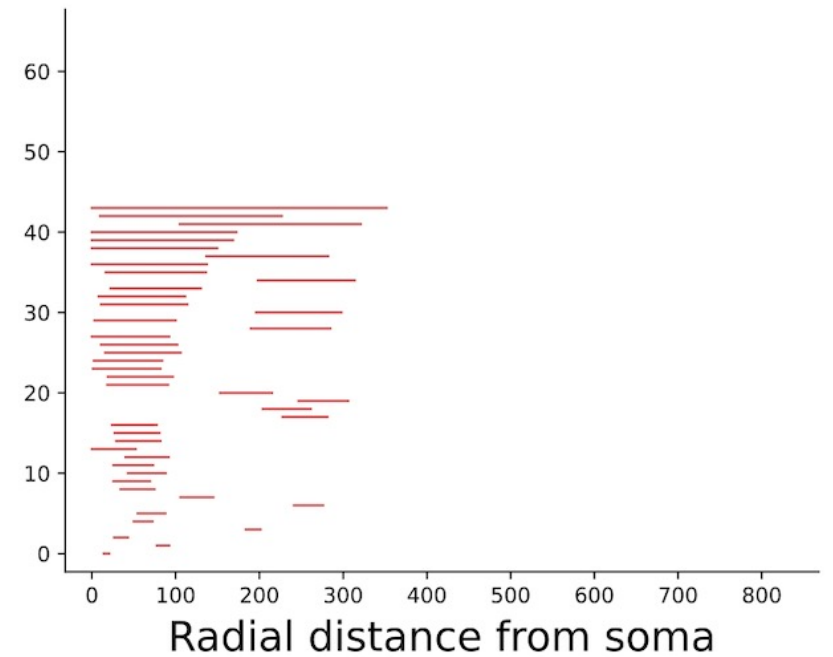
Topological barcode



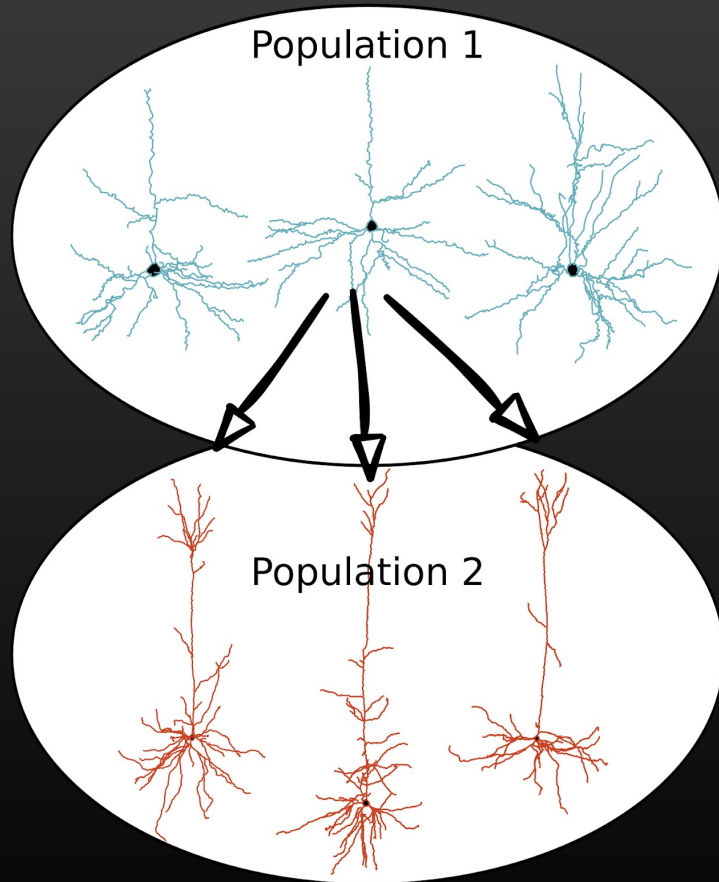
Topological difference (500um)



Topological barcode



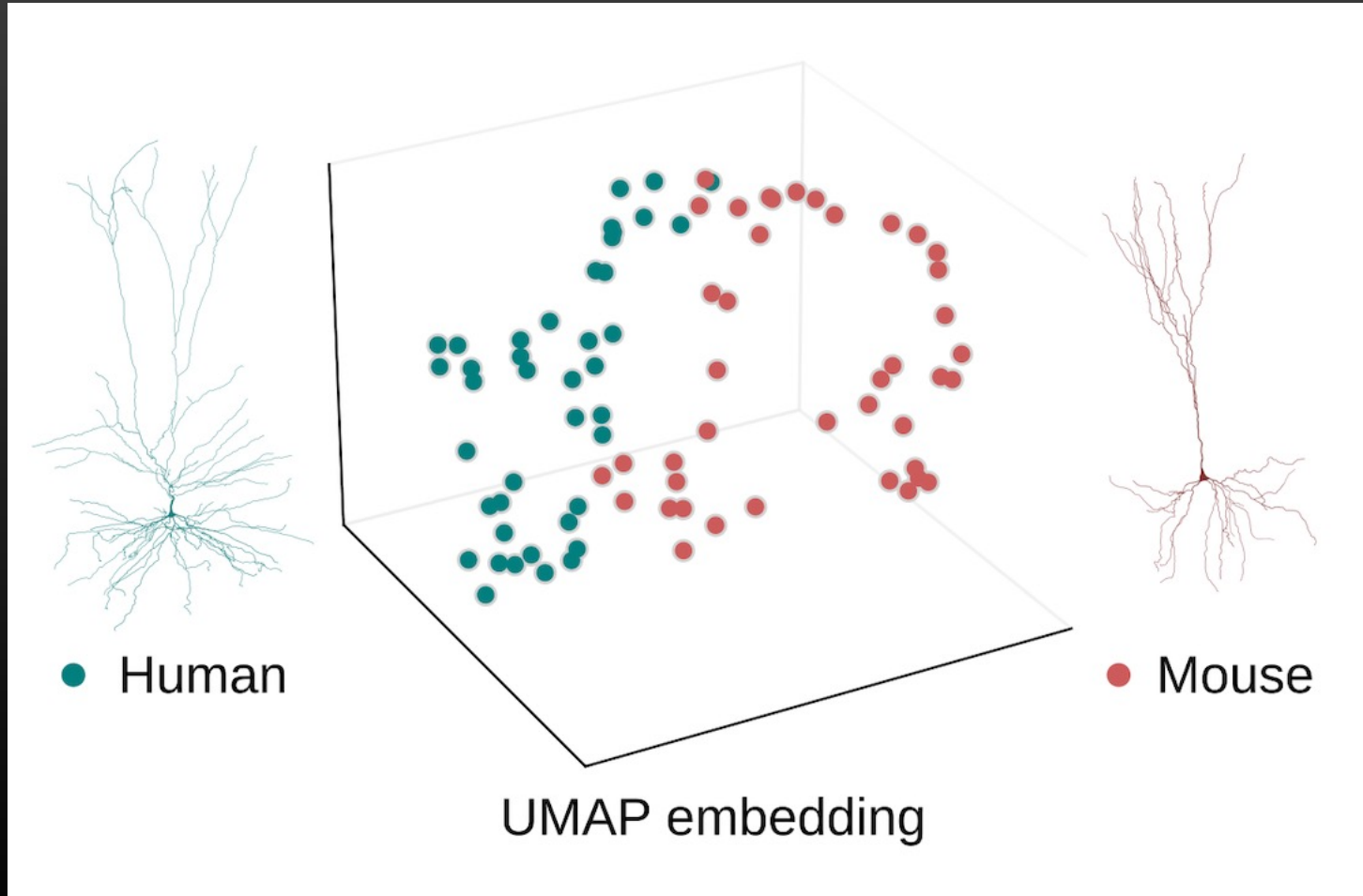
# Population comparison



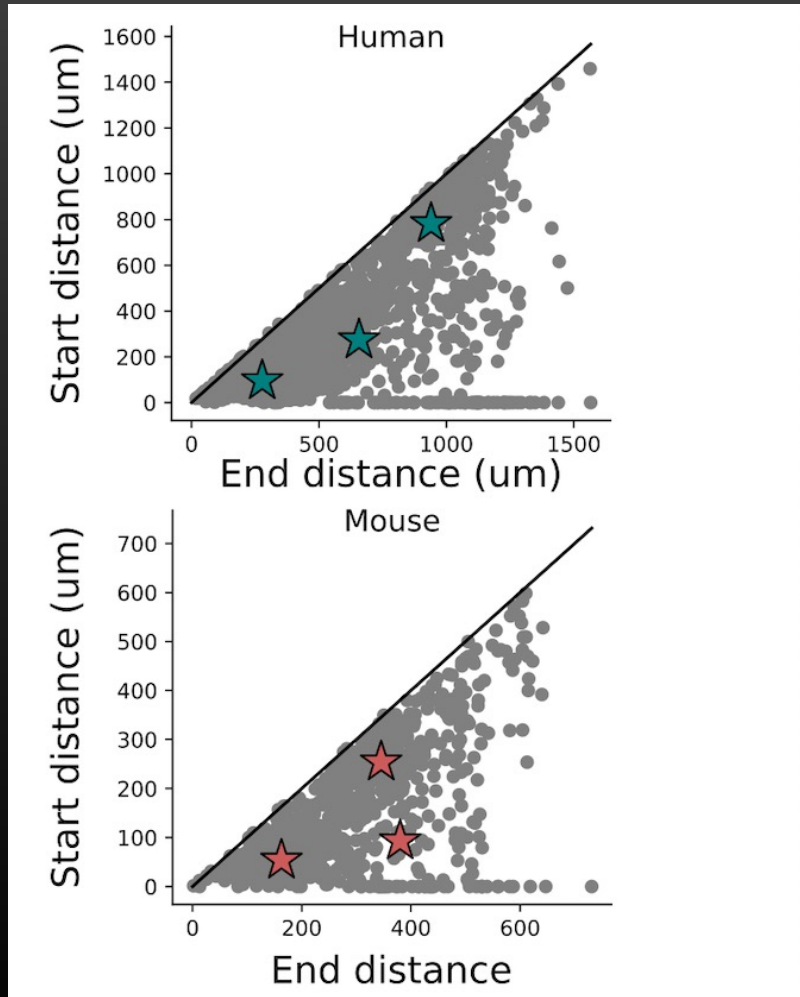
- Match neurons in the two populations with similar properties, e.g., cortical depth
- Compare important features (e.g., morphometrics, TMD) of matched neurons

# Topology distinguishes mouse and human populations

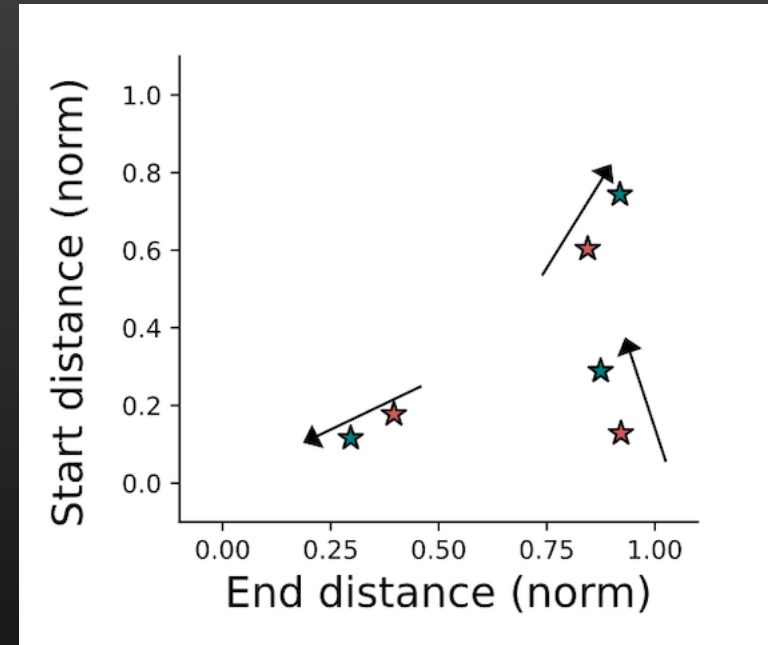
(Human neurons rescaled to enable comparison.)



# The insufficiency of rescaling



Population persistence diagrams



Optimal transformation of the  
Gaussian kernels

ii) Comparison of networks



Connections with direction!

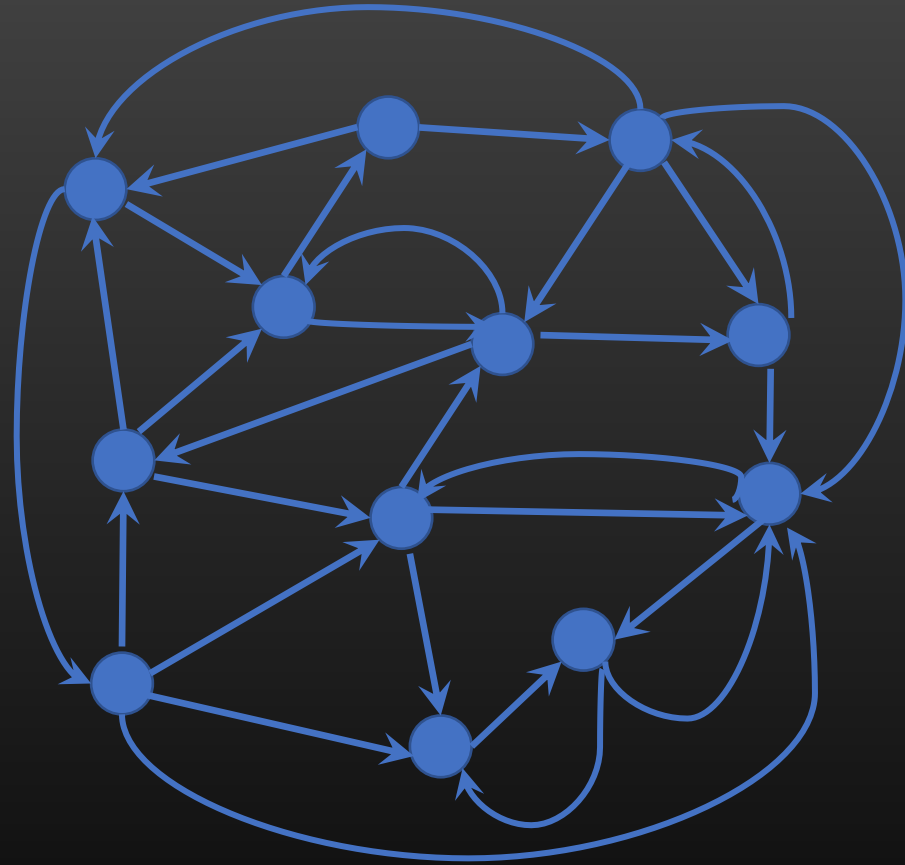






Connections with direction!

Represent the circuit by a **digraph**.



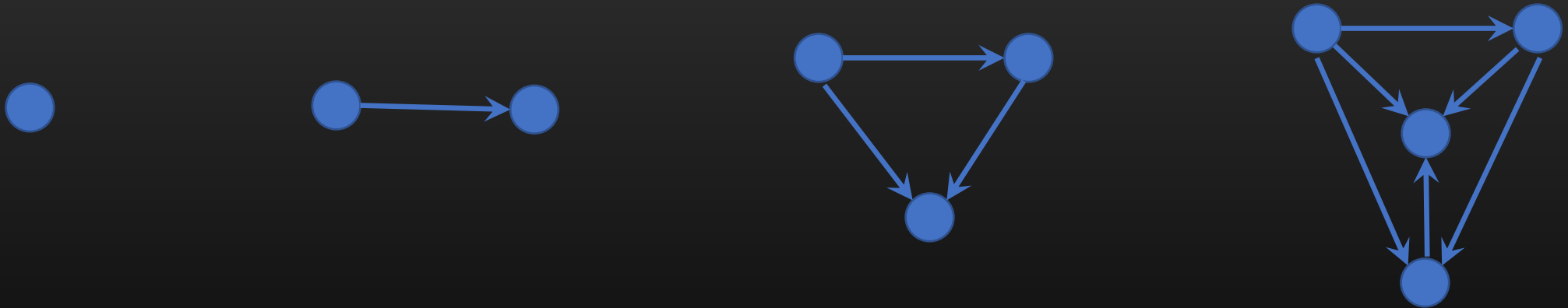
How to analyze and characterize the structure of a complex digraph?

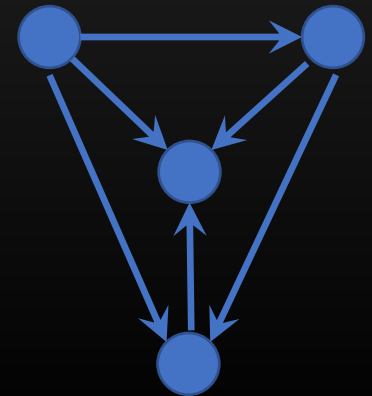
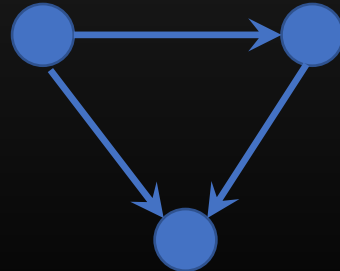
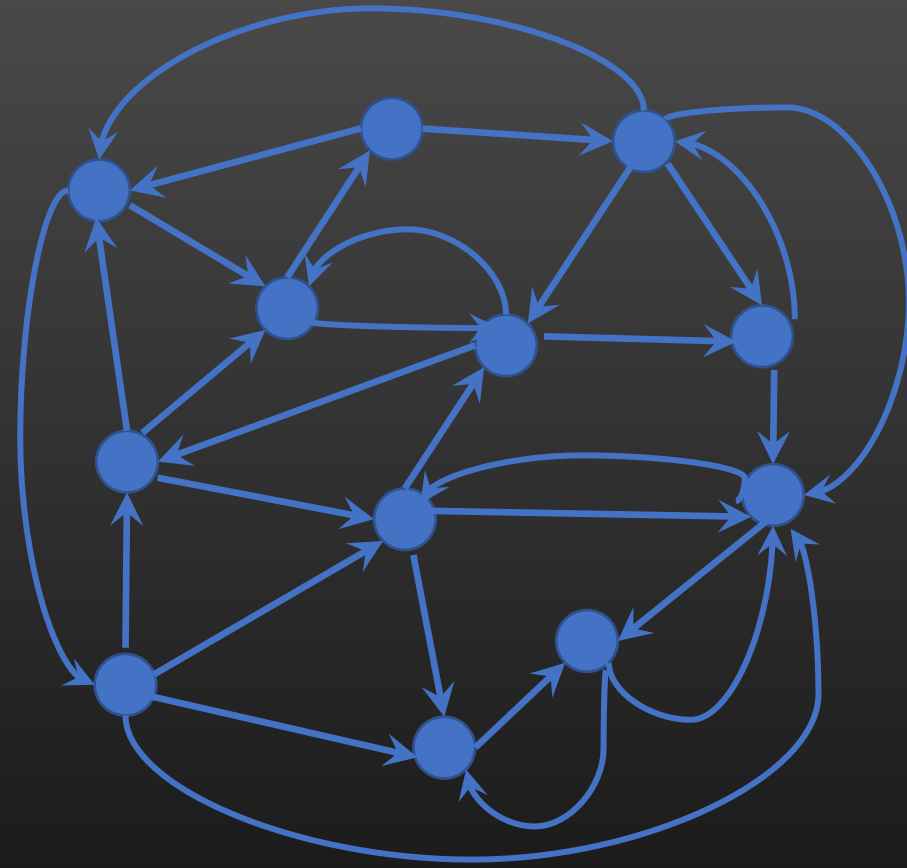
# Topological network analysis

- For each type of network (undirected/directed/weighted...), choose an appropriate family of **significant subnetworks** (e.g., motifs, graphlets) to study.
- The numbers of different types of significant subnetworks in a given network provide important **local information** about the network.
- Quantify how the significant subnetworks overlap in the network to obtain important **global information**.

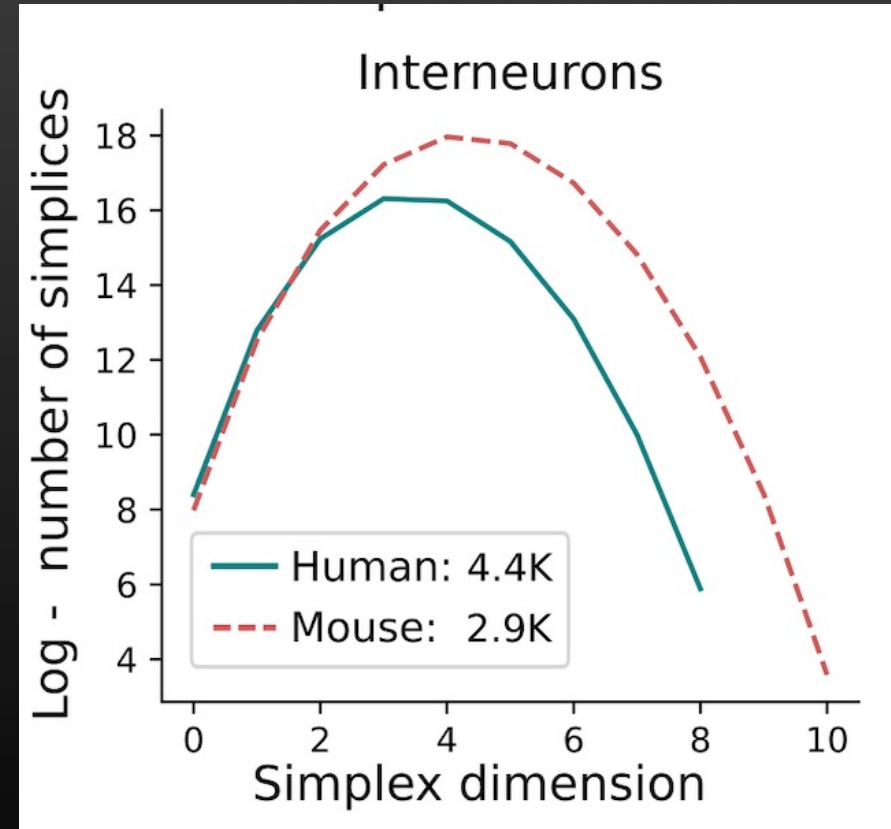
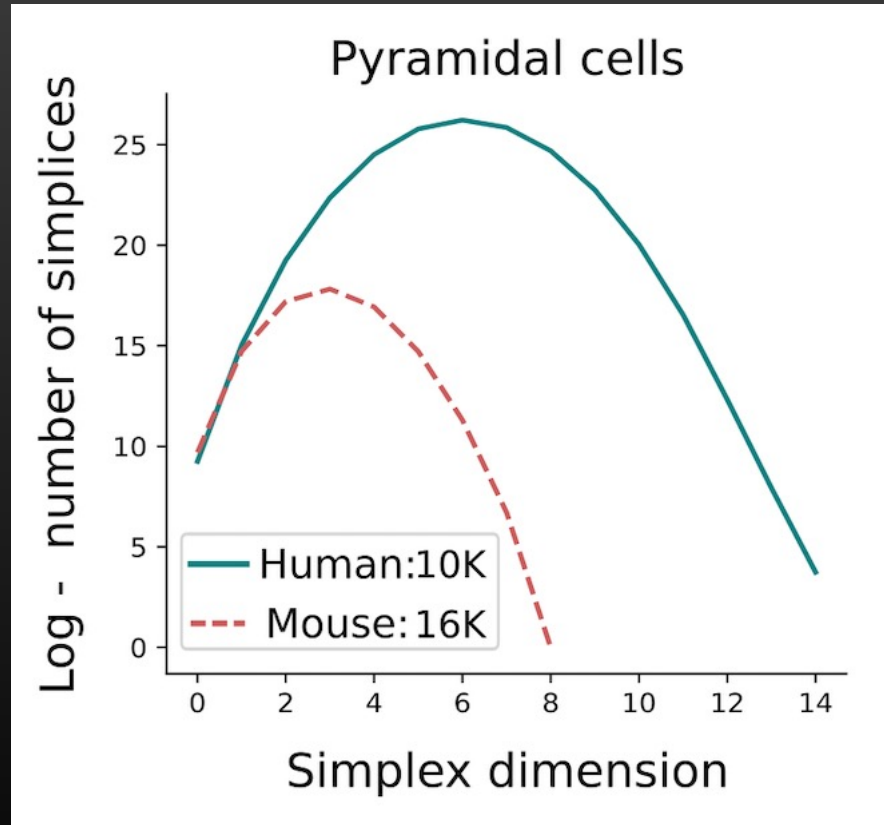
# Topological network analysis

Let  $\mathcal{G}$  be a directed graph. A **directed  $n$ -simplex** of  $\mathcal{G}$  is a complete, acyclic subgraph on  $n+1$  vertices of  $\mathcal{G}$ .



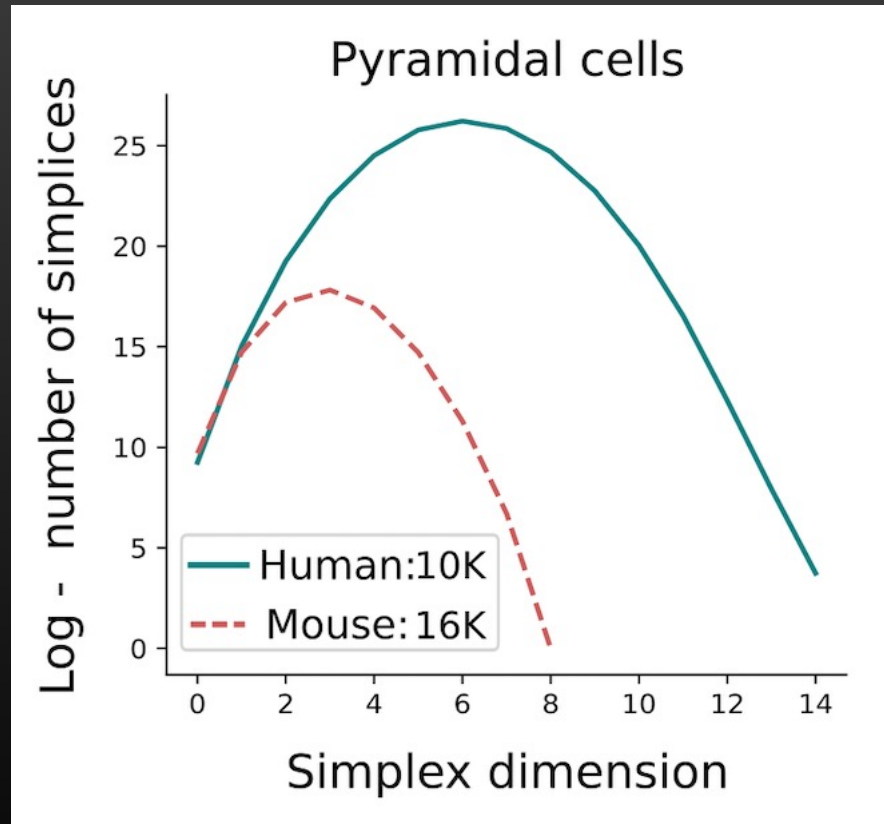


# Simplicial comparison of connectomes



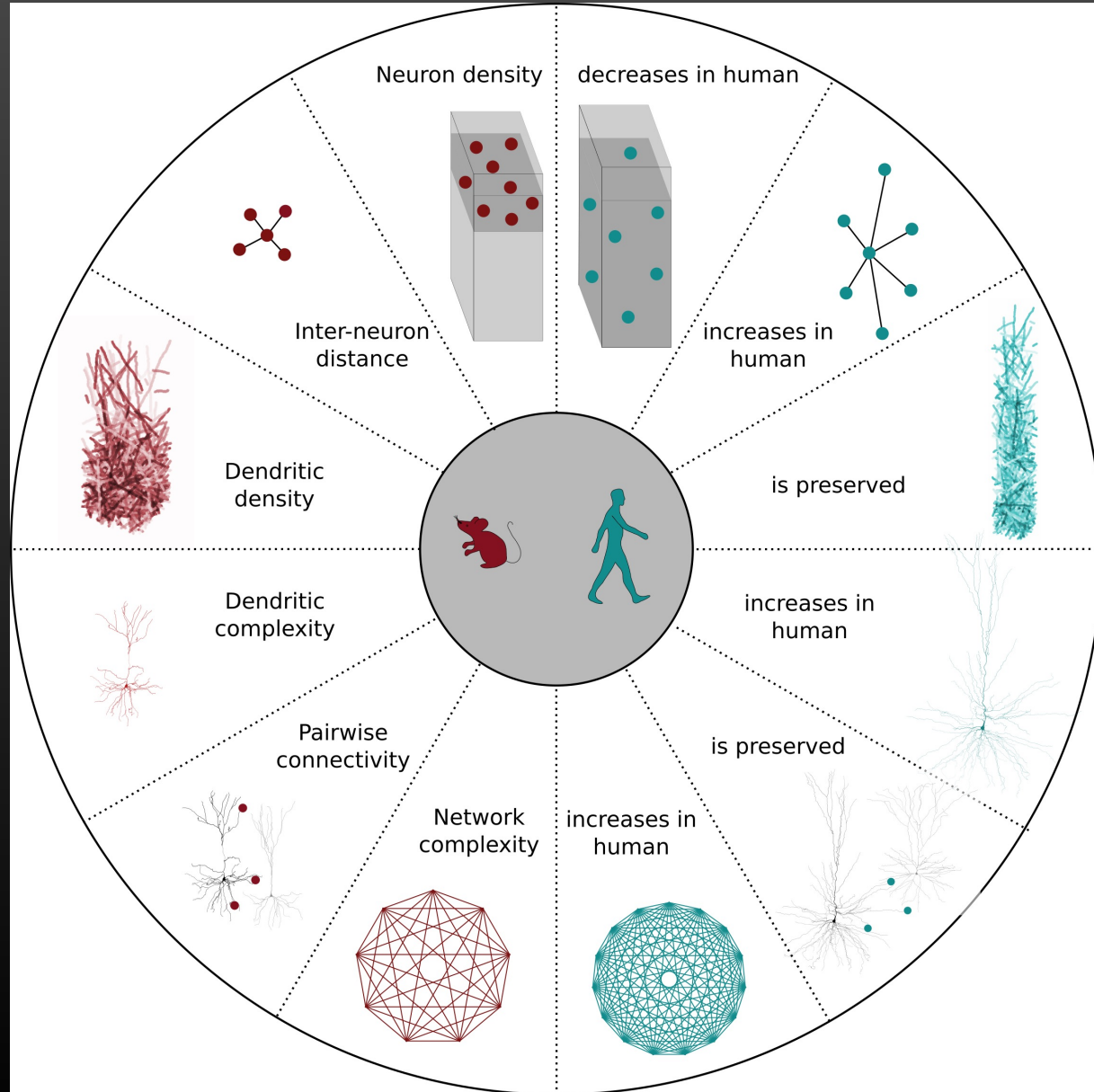
(Reconstructed volumes of 1mm x 1mm x layer thickness, supposing that 50% of appositions give rise to synapses.)

# Simplicial comparison of connectomes



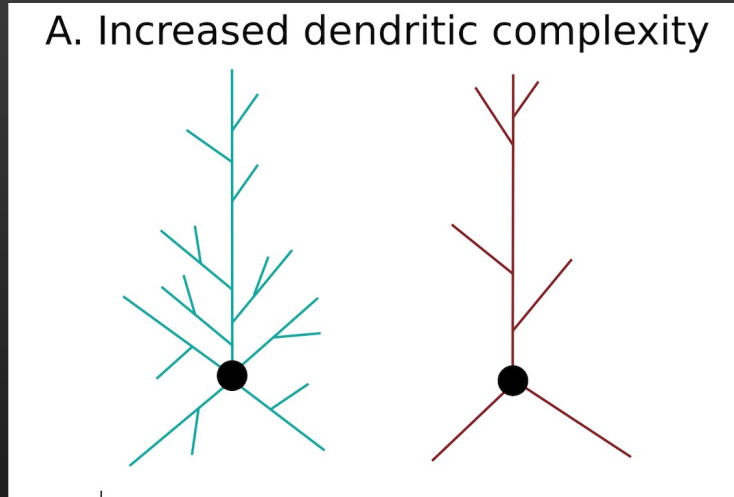
**Conclusion:** The greater complexity of the branching structure of human pyramidal dendrites more than compensates for the lower neuron density in human cortex, leading to substantially more complex network structure.

# Summary



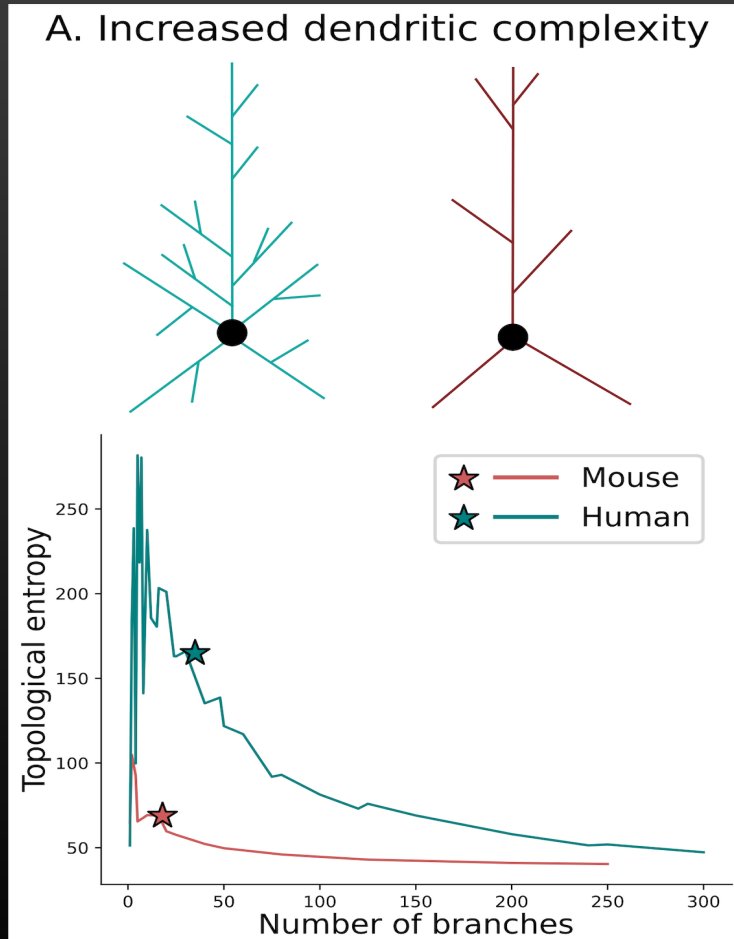


# Possible functional implications?



Why has the human brain evolved to prioritize complexity of individual neurons?

# Possible functional implications?

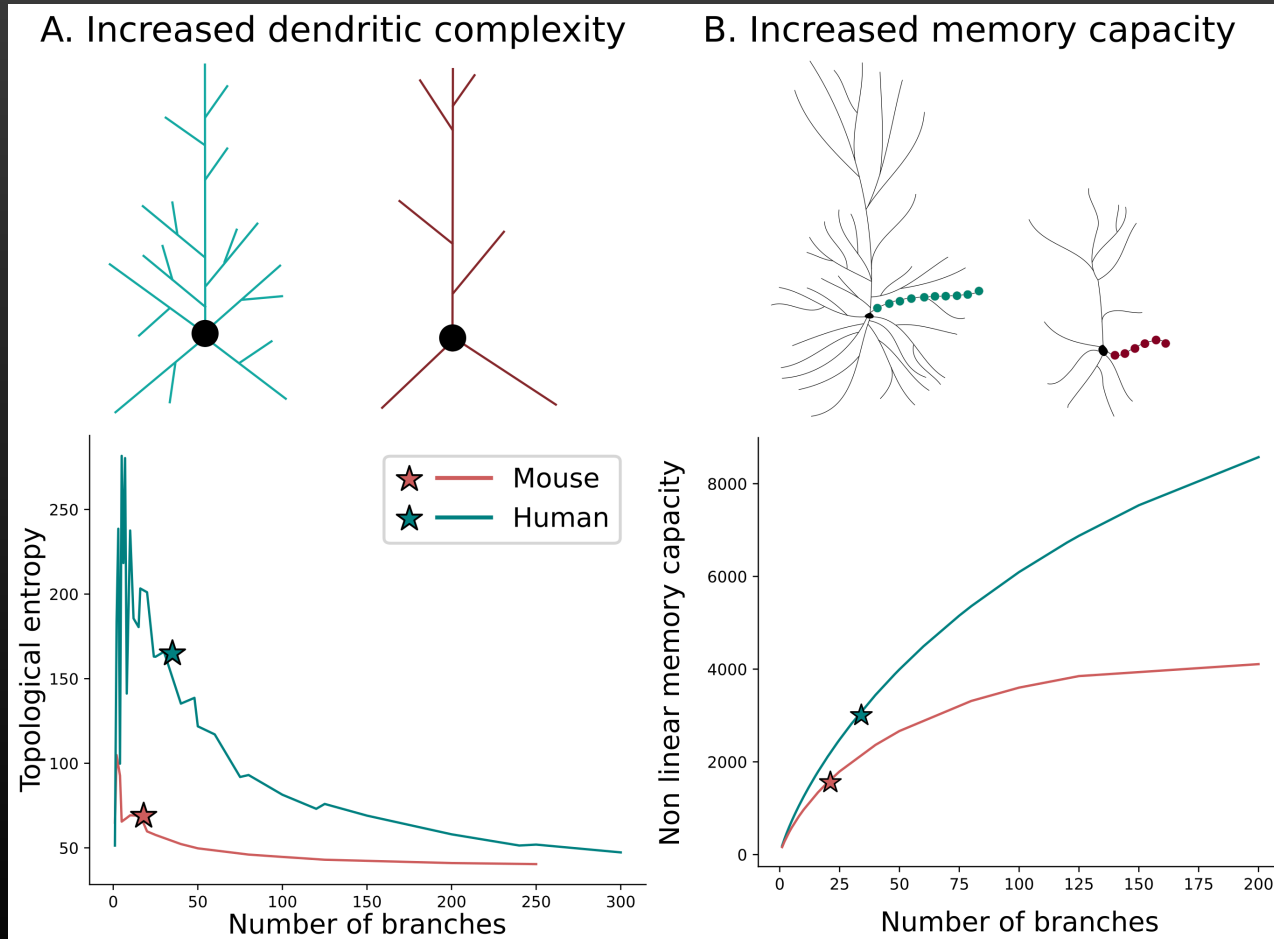


Topological entropy

$$E(PB) = - \sum_i^N \frac{l_i}{L} \cdot \log\left(\frac{l_i}{L}\right)$$

Chintakunta et al. (2015)

# Possible functional implications?



Topological entropy

$$E(PB) = - \sum_i^N \frac{l_i}{L} \cdot \log\left(\frac{l_i}{L}\right)$$

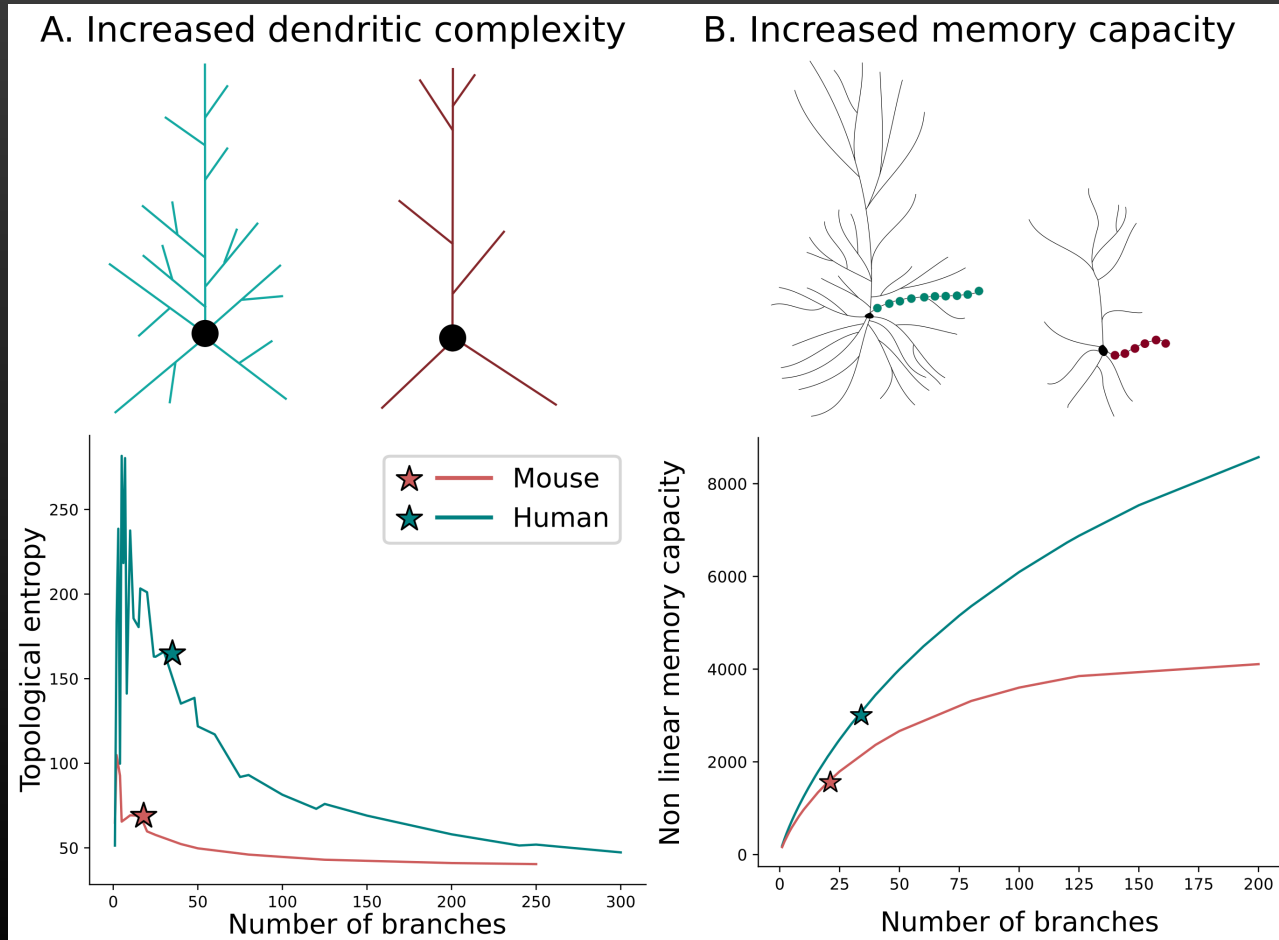
Chintakunta et al. (2015)

Nonlinear memory capacity

$$C_N = 2 \log_2 \left( \binom{k+d-1}{k} + m - 1 \right)$$

Poirazi and Mel (2001)

# Possible functional implications?



Memory capacity of individual neurons



Simplicial complexity of the connectome



Computational power of the connectome



*Thank you!*