Adaptive Approximation of Persistent Homology

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September 21st 2023
4th Workshop on Topological Methods in Data Analysis

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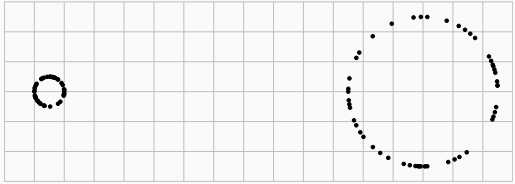
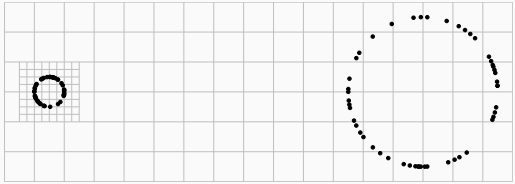
Adaptive Subsampling

Instead of this...

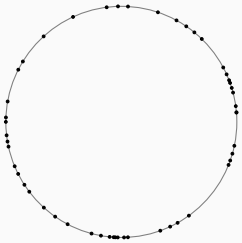


...we want something like this:



We need to decide how many points to keep in each area...

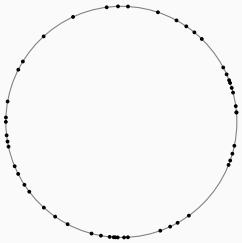
Example: Subsampling on Smooth Manifolds



Idea: Use manifold's curvature/folding to determine (local) sampling density.

Example: Subsampling on Smooth Manifolds

Use (an estimate of) the *local feature size* for adaptive sampling [ACK01, FR02]:



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Higher values for $\mathsf{lfs}(x)$ $\leftrightarrow$ Lower curvature at x

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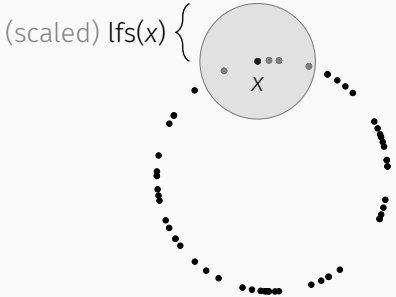

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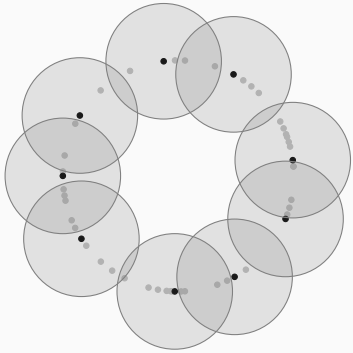

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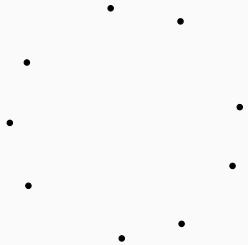
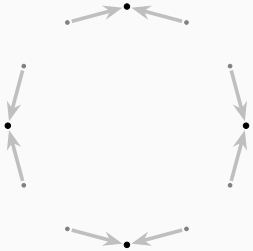


A Filtration on the Coarsened Subsample

To calculate the distance between two remaining points, we consider their clusters:



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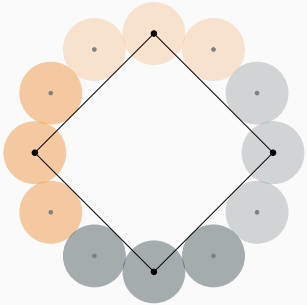

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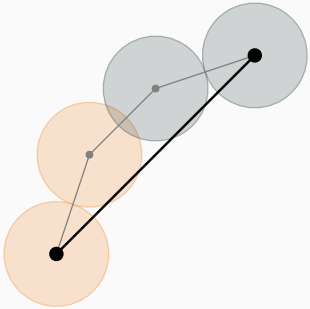
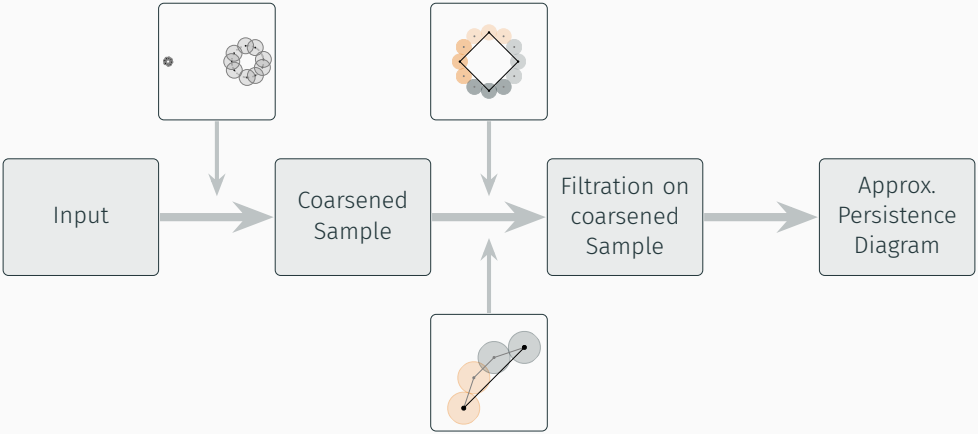


Edges in our filtration

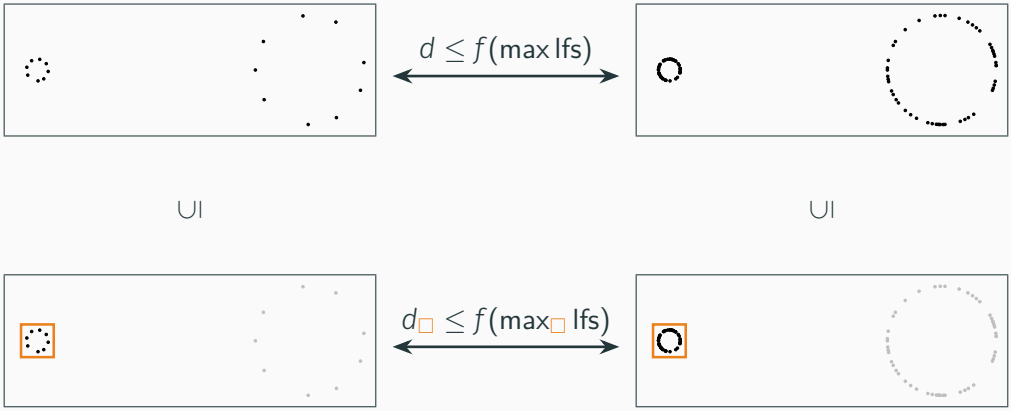
model

paths in the original filtration.

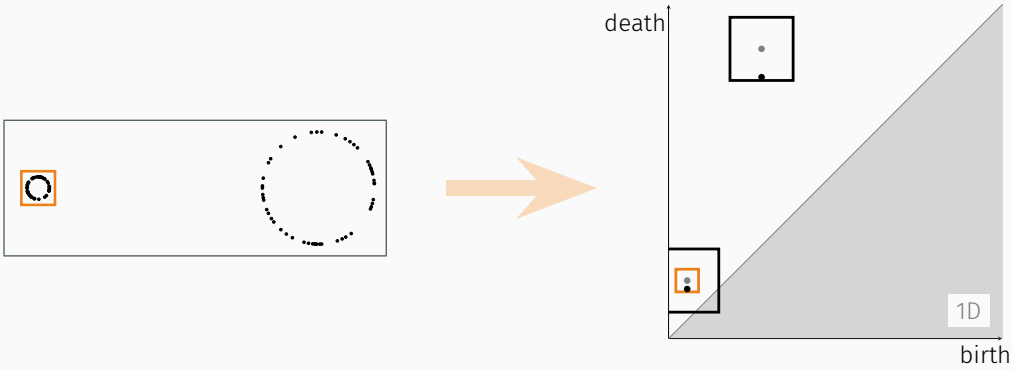
The Complete Algorithm



Adaptive Approximations

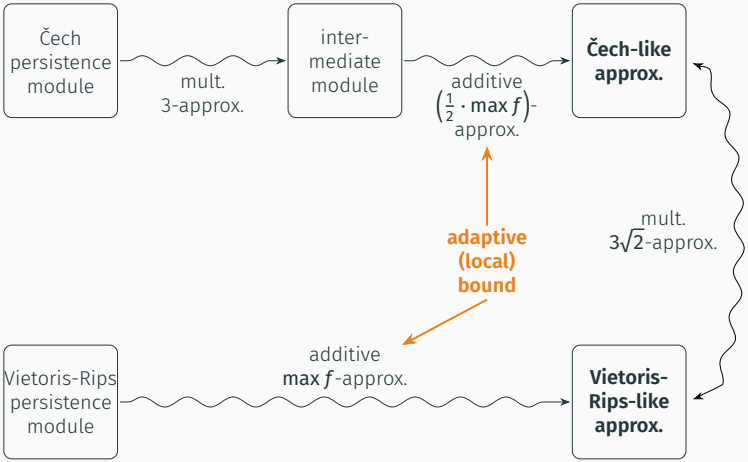


Adaptive Approximations



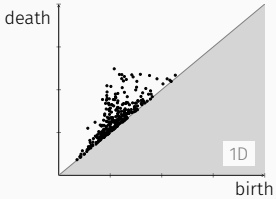
"The approximation quality for each feature depends only on the points on which certain representative cycles of the persistent homology class are defined."

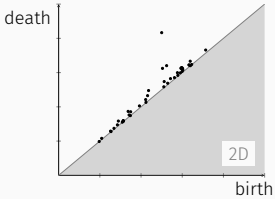
What We Can Prove



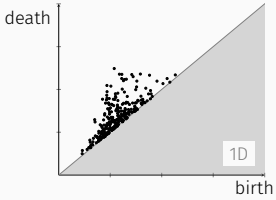
Experimental Results [OPT+17]

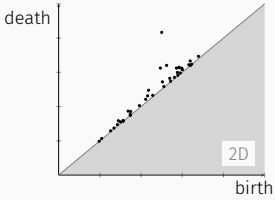
Persistence Diagrams:





Approximation in 15% of the original calculation time:





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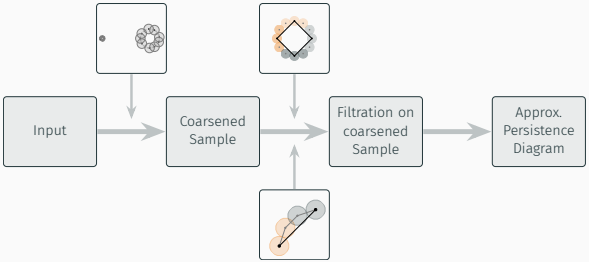
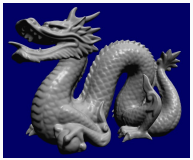
Included in the full version:

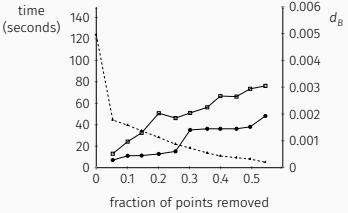
- Proofs (a lot of them)
- Empirical evaluation [OPT+17, Sta]

Future work:

- Finding interesting functions to use other than the *lfs*







Nina Amenta, Sunghee Choi, and Ravi Krishna Kolluri.
The power crust, unions of balls, and the medial axis transform.
Comput. Geom., 19(2-3):127–153, 2001.

Stefan Funke and Edgar A. Ramos.
Smooth-surface reconstruction in near-linear time.
In David Eppstein, editor, *Proceedings of the Thirteenth Annual ACM-SIAM Symposium on Discrete Algorithms, January 6-8, 2002, San Francisco, CA, USA*, pages 781–790. ACM/SIAM, 2002.

- Nina Otter, Mason A. Porter, Ulrike Tillmann, Peter Grindrod, and Heather A. Harrington.
  A roadmap for the computation of persistent homology.
  EPJ Data Sci., 6(1):17, 2017.
- Stanford University Computer Graphics Laboratory.
  The stanford 3d scanning repository.