TOPOLOGICAL TECHNIQUES TO QUANTIFY BIOLOGICAL PATTERN FORMATION

Daniel Tolosa Purdue University dtolosav@purdue.edu

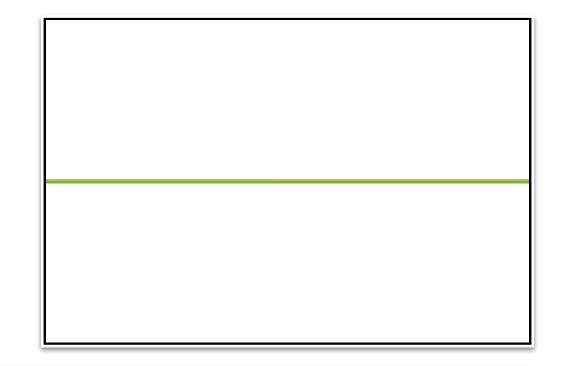
> **PURDUE** UNIVERSITY

{Peter-verreussel, iStock, pbs.org/wgbh/nova/article/zebrafish-sleep/}

#### Zebrafish (Danio rerio)

Wild type: striped pattern Mutants: variety of patterns







## **BIOLOGICAL BACKGROUND**

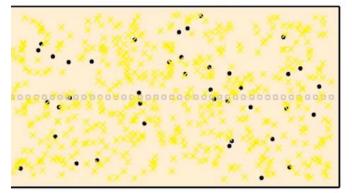
{Fish images: https://www.deccanherald.com/spectrum/science/zebrafish-emerging-research-model-671035.html, Frohnhöfer et al. Biol Open 2016, Irion et al. eLife 2014, Fadeev et al. eLife 2015, Singh et al. Curr Biol 2015, Courtesy of Uwe Irion & Christiane Nüsslein-Volhard, Video: Yamaguchi et al. PNAS 2007}

- Independent cells interacting in the plane
  - Three cell types, two of them with two subtypes, for a total of five types

$$\bigoplus_{M_i} \begin{array}{c} \bigstar \\ X_i^d \end{array} \\ X_j^\ell \end{array} \begin{array}{c} \Box_{I_j^\ell} \\ \nabla_{I_j^d} \end{array}$$

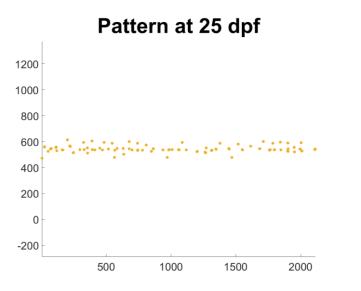
- Cell movement by differential equations
- Stochastic rules for other cell interactions

Stage PB  $\approx$ 21 dpf

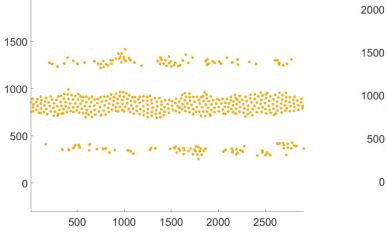


## AGENT-BASED MODEL OF PIGMENT CELL INTERACTIONS

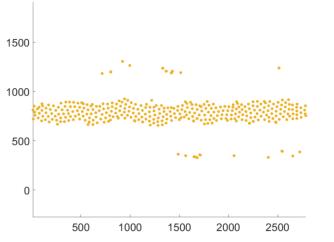
{Model: Iridophores as a source of robustness in zebrafish stripes and variability in Danio patterns, Volkening, Sandstede, Nature Commun 2018}



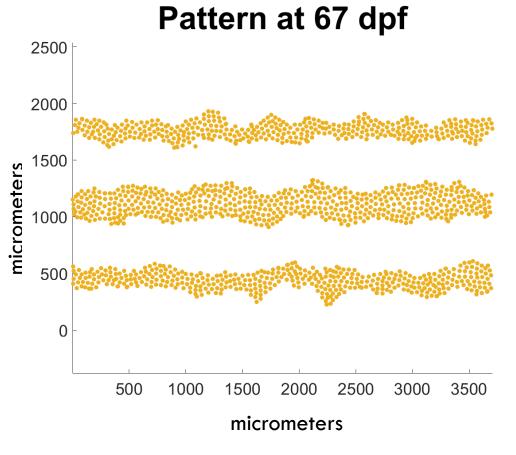
Pattern at 46 dpf



#### Pattern at 43 dpf

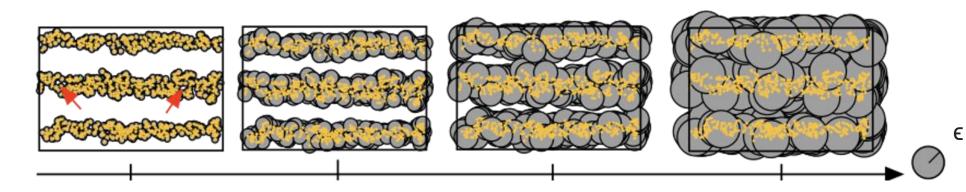


Pattern at 53 dpf

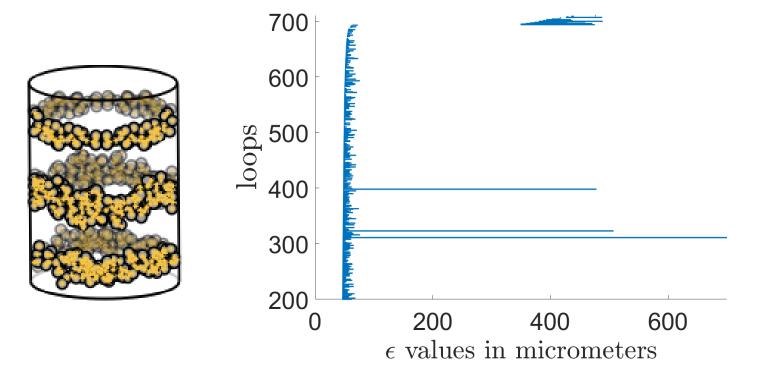


How many gold stripes are present at each time?

#### **CHALLENGE: QUANTIFYING PATTERN FORMATION IN TIME**



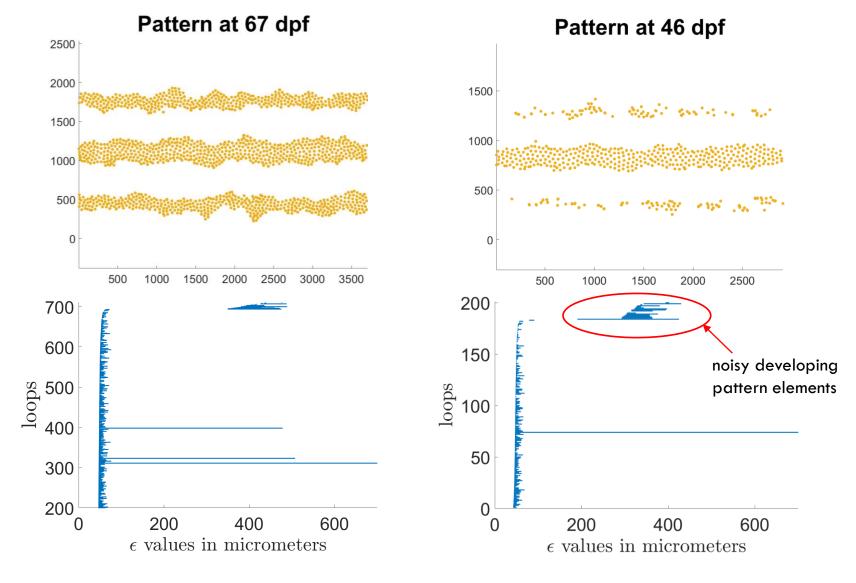
- Periodic domain
- Persistence = length of bar
- Loops with long bars are considered stripes
- McGuirl et al. used one persistence threshold: T<sup>1</sup> = 200 µm (specified a priori) to distinguish between noise and stripes



### **PRIOR WORK: PERSISTENT HOMOLOGY OF ZEBRAFISH**

{Reference: Topological data analysis of zebrafish patterns, McGuirl, Volkening, Sandstede, PNAS, 2020}

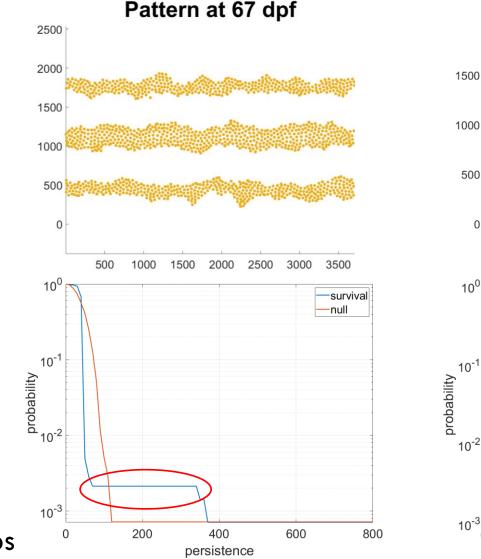
- Challenge: not flexible or applicable to all timepoints in zebrafish development
- How can we automatically choose a threshold that is dependent on the pattern alone?

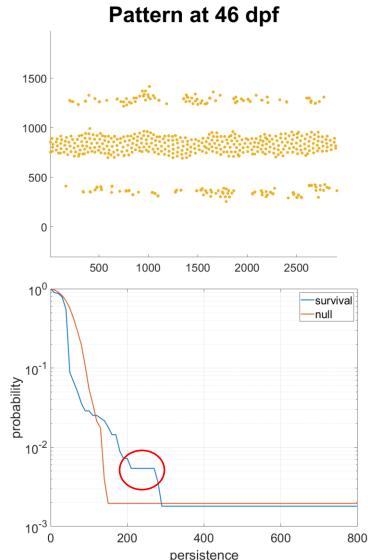


#### LIMITATIONS: DEPENDENT ON HYPERPARAMETERS AND DEVELOPED FOR FULLY FORMED PATTERNS

Ciocanel et al. used survival functions to identify meaningful loops in data on intracellular transport. We adapt this approach, and use survival functions to identify persistence thresholds for fish patterns

<u>Survival function S</u>: S(p) is the probability of randomly choosing a loop with persistence greater than or equal to p, from the set of loops



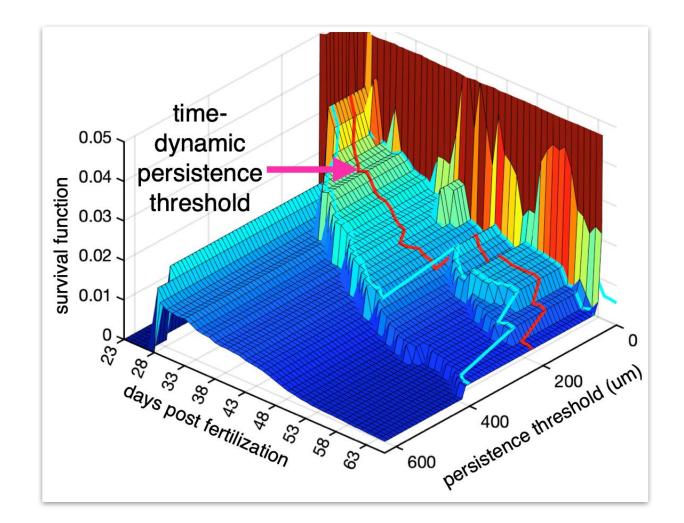


#### **OUR APPROACH: APPLYING SURVIVAL FUNCTIONS**

{Reference: Topological Data Analysis Approaches to Uncovering the Timing of Ring Structure Onset in Filamentous Networks, Ciocanel, Juenemann, Dawes, McKinley, Bull Math Biol., 2021}

#### Choosing a time-dependent theshold:

- Identify first plateau (at least 30 μm wide) at each timepoint of development
- Persistence threshold is midpoint of plateau at each given time
- Full plateau width provides a measure of confidence in our threshold



## **OUR APPROACH: SURVIVAL FUNCTIONS IN TIME**

- We specify persistence thresholds for loops by survival method
- Stripe width is based drops in gold and black stripe counts
- Curviness involves clustering based on stripe count
- Spots count uses dim 0 persistence, with a threshold depending on mean cell-to-cell distance

≈ 21 dpf

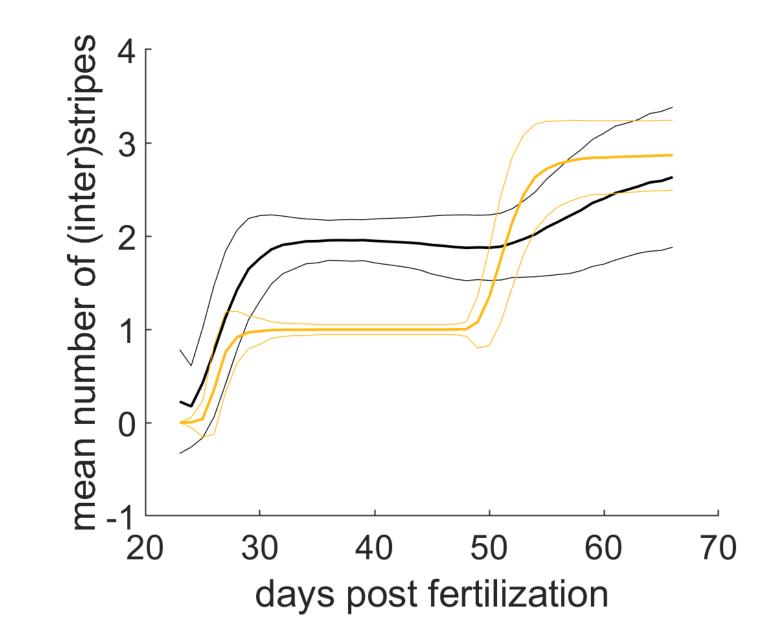
number of stripes = NaN, number of interstripes = NaN, number of gold spots = NaN mean stripe width = um, mean interstripe width = um mean curviness = NaN

# **RESULTS: APPLYING OUR METHODS TO ZEBRAFISH**

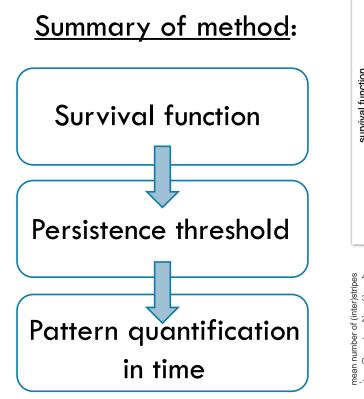
We ran statistical analyses on 1000 *in silico* patterns to understand large-scale behavior

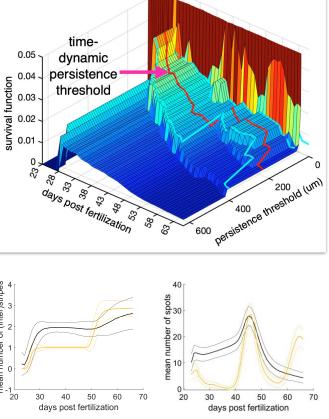
- Black cell (melanophore) stats in black
- Gold cell (xanthophore) stats in gold

RESULTS













Collaborator: Alexandria Volkening Ref: Tolosa, Volkening. In prep.

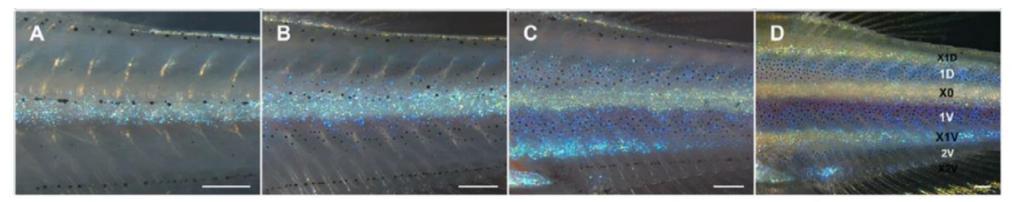


{Title: Zebrafish by Berliner Babylon, https://berlinerbabylon.com/.}

 Increasing the temperature of the environment can alter the striped pattern

				J+, 81 dpf 💻	J+, 91 dpf
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 Apply out methods to *in vivo* data.



### FUTURE WORK

{Figures: Iridophores as a source of robustness in zebrafish stripes and variability in Danio patterns, Volkening, Sandstede, Nature Commun 2018}